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ON THE LINEAR PRECODING OF NON-ORTHOGONAL STBC FOR CORRELATED MIMO CHANNEL

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ABSTRACT

In this paper, linear precoding for non-orthogonal space time block codes (STBC) is investigated. A theoretical model of spatial correlation with a Laplacian distribution of AOA is first derived. The design of the precoder is based on the choice of the codeword error matrix according to a criterion. We propose here a new criterion based on the system outage probability to select the suitable codeword error matrix allowing to move rapidly from one diversity order to the next. Codeword selection points out the importance of the determinant and the eigenvalues of the error matrices. The proposed method is applied to the non-orthogonal optimal STBC : 2×2 Golden Code and 4×4 Perfect Code.
Index Terms - MIMO Antennas, Space Time Coding, Correlation, Linear Precoding, Perfect Codes.

I. INTRODUCTION

The Multiple-Input Multiple-Output (MIMO) systems use multiple antennas at the transmission side as well as at the reception side which offers prospects to increase the capacity and to improve the quality of service for wireless communications. But, once faced to a real environment transmission, these systems suffer from the spatial correlation problem which induces a diversity decreasing and a performances degradation compared to a non-correlated environment.

Most of the space time code design criterions assume that antennas are uncorrelated and that MIMO channel matrix entries fade are independent which is not the case in practice. In fact, MIMO antennas are normally correlated, due to the lack of spacing between them and relating to the cone of arrival of the multiple paths. Thus, the high predicted spectral efficiency derived under the idealistic assumption that the channel matrix entries are independent complex gaussian variables, might be reduced on real channels. The effect of spatial correlation on the MIMO channel capacity has been addressed in [1]. Existing works on MIMO Correlation problem have focused on the effect of antennas separation, the angles spread of arrival, antennas arrangement (linear, hexagonal..), and antennas configurations (broadside, inline..). They proved that correlation problem becomes critical for small antennas separation and for small angle spread.

Several channel models are developed to find the correlation formulation. In [1], the one ring model was employed to determine the spatial fading correlation of the channel where the antennas array is surrounded by local scatters. In [2], a more general model was used and scatterers are present at both ends of the radio link and propagation is obstructed by a significant number of local scatterers. In the channel model, the selection of the angles of arrival (AoA) distribution describing the environment is crucial. In [3] and [4], a gaussian distribution was used. In the 3GPP MIMO channel [5], the laplacian distribution was adopted.

Recently works [6]-[7]-[8] proposed the use of linear precoder at the transmitter to reduce correlation effect when space time bloc codes are used. The precoder being calculated on the basis of knowledge of the matrix of correlation in order to minimize metric related to the error rate which is the pairwise error probability (PEP). Solution was derived in [6] for orthogonal-STBC (OSTBC) exploiting the fact that the hermitian product of the codeword error matrix is constant, which is not the case for non-orthogonal codes. Indeed, the precoder design is related to the hermitian product of only one codeword error matrix. Therefore, the performance could be enhanced of some code words but reduced for others. In [7], a solution was developed for the ABBA code, a Quasi-OSTBC. In [8], an approximated solution was given based on average design over all codeword error matrices.

In this paper, the 3GPP channel model [5] is addressed and its correlation matrix is developed. The general linear precoder solution proposed in [6] is used. A new criterion based on the outage probability to find a rule for selecting the suitable codeword error matrix for the precoder design is derived. The basic idea is to choose the error matrix corresponding the best system outage probability. This design is applied to the Golden Code, which is a non-orthogonal optiml STBC. [9].

This paper is organized as follows. Section II describes the adopted channel and the correlation matrix models. By the way, a theoretical correlation form of a Laplacian distribution of AoA is given. Section III derives the linear precoder solution based on the average PEP criterion. Non-orthogonal STBC constraint was explored and methods to face this problem are studied. Section IV presents the Golden Code and it was shown how the appropriate codeword error matrix selection is crucial. In section V, simulations results are provided and discussed.

II. Channel Model

A. General Description

Let's consider a MIMO channel with a linear array of n_T transmit and n_R receive antennas. The received signal is then defined as

$$\mathbf{Y} = \mathbf{HW} + \mathbf{B}, \tag{1}$$

where $\mathbf{H}$ is the $n_R \times n_T$ channel matrix, $\mathbf{Y}$ is the received signal corrupted by an additive white gaussian noise denoted $\mathbf{B}$ with covariance matrix $\sigma_b^2 \mathbf{I}_{n_R}$, $\mathbf{W}$ is an $n_T \times T$ codeword of the STBC and T is the temporal codelength. We consider a coherent system, where the channel matrix is supposed perfectly known at the receiver side.

Let's assume that transmit and receive correlations are separable [3]. In such a case, an $n_R \times n_R$ correlation coefficient matrix can be defined and denoted by $\mathbf{S}$, for the receive antennas and $n_T \times n_T$ correlation matrix denotes by $\mathbf{R}$ for the transmit antennas. Thus the correlated MIMO channel matrix can be represented as

$$\mathbf{H} = \mathbf{S}^{1/2}\mathbf{H}_w\mathbf{R}^{1/2}, \tag{2}$$

where $\mathbf{H}_w$ is an $n_R \times n_T$ i.i.d complex gaussian matrix with zero mean and unit variance. Let $\mathbf{S} = \mathbf{S}^{1/2}\mathbf{S}^{1/2^H}$ and $\mathbf{R} = \mathbf{R}^{1/2}\mathbf{R}^{1/2^H}$.

In the following, only correlation at the transmitter will be taken into account. Thus, the channel model in this environment is reduced to

$$\mathbf{H} = \mathbf{H}_w\mathbf{R}^{1/2}. \tag{3}$$

Only the correlation matrix is assumed to be known at the transmitter side.

B. Correlation Model

Let's assume ρ_{ik} the correlation coefficient between the i^{th} and the k^{th} transmitter antenna array elements. They are obtained using the components of the channel matrix

$$\rho_{ik} = \frac{1}{\sqrt{\beta_i\beta_k}} r_{ik}, \tag{4}$$

where $r_{ik} = E\left[\sum_j h_{ij}h_{kj}^*\right]$ and $\beta_i = r_{ii}$ is the normalized average power in the i^{th} receive branch.

The theoretical adopted form of r_{ik} is given by [3]

$$r_{ik} = \begin{cases} \int_{-\sigma/2}^{\sigma/2} \exp^{-j2\pi\frac{d_{ik}\sin\theta}{\lambda}} p(\theta)d\theta & i \neq k \\ 1 & i = k \end{cases} \tag{5}$$

here $p(\theta)$ is the pdf of the direction of arrival, σ is the receive angular spread and $d_{ik} = (i-k)x$, where x is the separation between two consecutive antenna elements. The Power Azimuth Spectrum (PAS) of a path arriving at the MS is modeled as a Laplacian distribution for the 3GPP MIMO channel [5]. For an incoming AoA $\bar{\theta}$ and RMS angle spread σ , the MS per-path Laplacian PAS value at angle θ is given by

$$P(\theta, \sigma, \bar{\theta}) = N_0 exp\left[\frac{-\sqrt{2}|\theta - \bar{\theta}|}{\sigma}\right], \tag{6}$$

where N_0 is the normalization constant and is given by

$$N_0^{-1} = \int_{-\infty}^{+\infty} exp\left[\frac{-\sqrt{2}|\theta - \bar{\theta}|}{\sigma}\right] d\theta. \tag{7}$$

In (6), θ and $\bar{\theta}$ are given with respect to boresight of the antenna elements [11]. To establish theoretical form of correlation with laplacian distribution, and in similar way to [12], one can prove that

$$\begin{aligned} \rho(x, \bar{\theta}, \sigma) &= E\left[e^{-j\frac{2\pi}{\lambda}x\sin(\theta+\bar{\theta})}\right] \\ &= N_0 \int_{-\infty}^{+\infty} e^{-j\frac{2\pi}{\lambda}x\sin(\theta+\bar{\theta})} e^{-\sqrt{2}\frac{|\theta|}{\sigma}} d\theta \end{aligned} \tag{8}$$

If σ is small the approximation $\sin(\theta + \bar{\theta}) \simeq \sin(\bar{\theta}) + \theta\cos(\bar{\theta})$ can be made and one can get

$$\rho(x, \bar{\theta}, \sigma) = \frac{2e^{-j\frac{2\pi}{\lambda}x\sin\bar{\theta}}}{2 + [\frac{2\pi}{\lambda}\sigma x\cos\bar{\theta}]^2}. \tag{9}$$

III. Linear Precoding

A. Pair-wise Error Probability

One way to resolve the correlation problem is the use of linear precoder which operates on the space-time coded symbols. The design of this precoder assumes the knowledge of the transmit correlation. The PEP is the error probability of deciding the codeword $\mathbf{W}_k$ instead of the transmitted codeword $\mathbf{W}_j$. The codeword $\mathbf{W}_j$ estimation is given by an ML minimization. Let's denote $\mathbf{E}$ the error matrix defined by $\mathbf{E} = \mathbf{W}_j - \mathbf{W}_k$ and $\mathbf{P}$ the linear precoder to be added at the transmitter. The precoder includes linear transformations to adapt codewords to the available channel. The received signal is then defined as

$$\mathbf{Y} = \mathbf{HPW} + \mathbf{B}. \tag{10}$$

Following steps in [10], PEP between $\mathbf{W}_k$ and $\mathbf{W}_j$ can be upper bounded by

$$\overline{PEP}(\mathbf{E}) \leq \left[det\left(\mathbf{I} + \frac{E_s}{4\sigma_b^2}\mathbf{R}^{1/2}\mathbf{PEE}^H\mathbf{P}^H\mathbf{R}^{1/2^H}\right)\right]^{-n_R}, \tag{11}$$

where E_s is the symbol energy . The optimal precoder that minimizes the PEP is the solution of the following optimization problem [6]

$$\max_P J = det\left(\mathbf{I} + \frac{E_s}{4\sigma_b^2}\mathbf{R}^{1/2}\mathbf{PEE}^H\mathbf{P}^H\mathbf{R}^{1/2^H}\right) \tag{12}$$

subject to :

$$Tr\left(\mathbf{PP}^H\right) = p_0, \tag{13}$$

where (13) constrains the total power across n_T transmit antennas to p_0. This problem has a water-filling solution. In fact, by defining the singular value decomposition of $\mathbf{R}^{1/2} = \mathbf{U}_R\Lambda_R\mathbf{V}_R^H$ and $\mathbf{EE}^H = \mathbf{V}_E\Lambda_E\mathbf{V}_E^H$, the solution is [6]

$$\mathbf{P} = \mathbf{V}_R\Phi\mathbf{V}_E^H, \tag{14}$$

with

$$\Phi^2 = \left[\mu \mathbf{I} - \left[\frac{E_s}{4\sigma_b^2}\right]^{-1} \Lambda_R^{-2} \Lambda_E^{-1}\right]_+ , \quad (15)$$

where μ is the water-filling constant and $[.]_+$ means $max(.,0)$

From this result, it follows that in the presence of transmit correlation, the geometry of the codeword difference matrices $\mathbf{E}$ could have deep impact on the performance.

B. Orthogonal Codes Precoding

Orthogonal space-time codes have $\mathbf{E}\mathbf{E}^H = \beta \mathbf{I}$, in which case $\Lambda_E = \beta \mathbf{I}$ and $\mathbf{V}_E = \mathbf{I}$, where β is a scalar. The rotation matrix $\mathbf{V}_R$ ensures that the optimal precoder pours power only over eigenmodes of $\mathbf{R}$. Then, the precoder solution can be simplified to get this form

$$\mathbf{P} = \mathbf{V}_R \Phi \quad (16)$$

$$\Phi^2 = \left[\mu \mathbf{I} - \left[\frac{\beta E_s}{4\sigma^2}\right]^{-1} \Lambda_R^{-2}\right]_+ \quad (17)$$

In such a scenario, the optimal precoder can be considered as a statistical eigenbeamformer. One can remark that precoder (16) doesn't depend on couples of codewords error $\mathbf{E}$. The precoder enhancement for an orthogonal code can be observed on the Alamouti code. This code is optimal for $n_T = 2$, $n_R = 1$. Figure 1 shows the bit error rate (BER) for the i.i.d channel coefficients case, the case with correlation and with precoding as a function of the SNR, the correlation term is equal to 0.9. The Precoding gain is obtained spatially at low SNR where the transmission uses only one eigen mode. But for high SNR, improvements are not distinguished because of the waterfilling behavior.

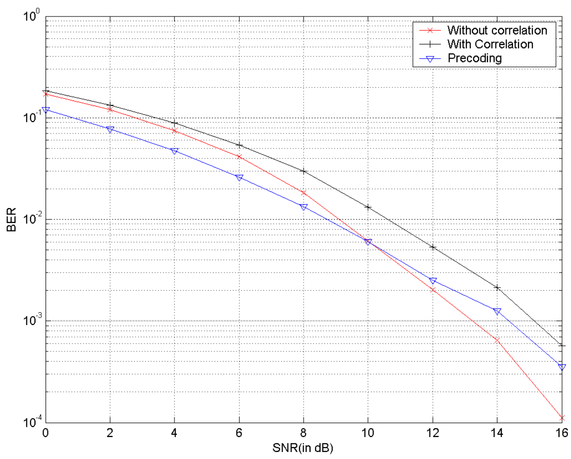

Figure 1: Alamouti in Correlated Channel with $\rho = 0.9$ for QPSK

C. Non-orthogonal Codes Precoding

For the non-orthogonal codes the property $\mathbf{E}\mathbf{E}^H = \beta \mathbf{I}$ is not verified. The power allocation on the eigenmodes of $\mathbf{R}$ given by the waterfilling policy, depends on the eigenvalues of $\mathbf{E}$ and $\mathbf{R}$. One can look then for the effective minimum distance error matrix among all possible codewords and select codewords having the minimal determinant. This solution proposed in [6] gives as result many codewords verifying this condition.

The upper bound (11) depends on the codeword error matrix $\mathbf{E}\mathbf{E}^H$ and choosing one $\mathbf{E}$ can enhance the performance for the couple of codewords (by minimizing their PEP upperbound) but degrade other couples. In fact, and as explained in [7], if $\mathbf{E}_{min}$ is a code error matrix with minimum determinant, then $\mathbf{E}_{min}\mathbf{E}_{min}^H$ is not unique. For example, $\mathbf{E}_1$ and $\mathbf{E}_2$ can have the same minimum determinant but $\mathbf{E}_1\mathbf{E}_1^H$ and $\mathbf{E}_2\mathbf{E}_2^H$ are different. Minimizing the averaged PEP for one of them doesn't ensure the performance of the others and it can even degrade the precoder performance instead of improving it. To resolve this problem, two solutions already exist on the letterature, and we propose here a new one.

1) Medles method

Medles *et al.* define in [7] a set of non-zero code error matrices $\Omega = \{p, q : \mathbf{E}(p,q) \neq 0\}$.
Then, another set Ω_1 is defined as $\Omega_1 = \left\{\mathbf{A}/\mathbf{A} \leq \mathbf{E}\mathbf{E}^H, \forall \mathbf{E} \in \Omega\right\}$.
Where for $\mathbf{A}_1$ and $\mathbf{A}_2$ two $N \times N$ complex matrices : $\mathbf{A}_1 \leq \mathbf{A}_2$ if and only if for all $\mathbf{v} \in C^N$ $\mathbf{v}^H\mathbf{A}_1\mathbf{v} \leq \mathbf{v}^H\mathbf{A}_2\mathbf{v}$.
Let's $\Upsilon = sup\left(\Omega_1\right)$, the supreme matrix of the set Ω_1. Then, if Υ is non-zero, the property :

$$\mathbf{A}_1 \geq \mathbf{A}_2 \geq 0 \Rightarrow det\left(\mathbf{A}_1\right) \geq det\left(\mathbf{A}_2\right). \quad (18)$$

This property can be used to prove that Υ minimizes the upper bound of the PEP[7] .

2) Paulraj method

An average design solution was proposed in [8]. Instead of employing the worst case analysis where $\mathbf{E}$ corresponds the minimum distance over all pairs of codewords, another design based on the average distance over all pairs of codewords was considered. Denoting $\mathbf{G} = \mathbf{E}\mathbf{E}^H$, and in the average context, the average distance measure of $\mathbf{B}$ is

$$\bar{\mathbf{G}} = \frac{1}{T} E\left[(\mathbf{W} - \hat{\mathbf{W}})(\mathbf{W} - \hat{\mathbf{W}})^H\right], \quad (19)$$

where$E[.]$ means the expectation over all possible codeword pairs. $\bar{\mathbf{G}}$ represents the covariance of codeword error statistics over all codewords. The detection is done jointly over T symbol times.

The error bound is monotonic in $\mathbf{G}$ [8]. Compared to the minimum distance criterion the average distance $\bar{\mathbf{G}}$ gives a smaller value in the bound . Even if this does not guarantee a minimum precoding gain, it leads to a valid precoder. Often $\bar{\mathbf{G}}$ is a scaled identity matrix which is not the case for the minimum distance $\mathbf{G}$ with non-orthogonal STBC.

The advantage of this design is that complexity is reduced with a minimum gain. Even if precoding performances decreases, this method seems to be very useful specially for high constellation sizes where the search of the minimum codeword error matrix is too complex.

3) Proposed method

To select the codeword error matrix, the outage probability is studied. Let's consider the following linear system

$$\mathbf{Y} = \mathbf{HPMX} + \mathbf{B} \tag{20}$$

where $\mathbf{M}$ is the linear space time coding matrix, $\mathbf{X}$ is the constellation symbols matrix and $\mathbf{P}$ is the precoding conditioned on one fixed error matrix $\mathbf{E}$.

The outage probability is defined as

$$P_{out}(R) = P\left(C(\mathbf{H}) < R\right) \tag{21}$$

where the capacity is

$$C(\mathbf{H}) = \log_2\left[\det\left(I_n + \frac{E_s}{4\sigma^2}\mathbf{H}_w\mathbf{R}^{1/2}\mathbf{PMM}^H\mathbf{P}^H\mathbf{R}^{1/2^H}\mathbf{H}_w{}^H\right)\right]. \tag{22}$$

The outage probability represents the lower bound of error probability for the coding system. Increasing system diversity can reduce this lower bound providing a relevant enhancement in performance. For low SNR, the precoder (16) proceed as a beamformer which reduces enormously the system diversity. Thus, we need a new criterion to build $\mathbf{P}$, leading to a quick move to more diversity. One can select the codeword that verify criterion among the codewords with minimum determinant. The difference between the eigenvalues of the codeword should be the least possible. Thus, waterfilling will rapidly stop beamforming and will move to higher diversity.
Let's ϵ be the minimum determinant of $\mathbf{EE}^H$ and let

$$\Gamma = \left\{\mathbf{E}_i : \mathbf{E}_i \neq 0, \det\left(\mathbf{E}_i\mathbf{E}_i^H\right) = \epsilon\right\} \tag{23}$$

be the set of codeword error matrices verifying the minimum determinant ϵ. For each $\mathbf{E}_n \in \Gamma$, the hermitian matrix $\mathbf{E}_n\mathbf{E}_n^H$ has the following eigenvector decomposition $\mathbf{E}_n\mathbf{E}_n^H = \mathbf{V}_{E_n}\Lambda_{E_n}\mathbf{V}_{E_n}$ where $\Lambda_{E_n} = diag\left(\lambda_{n1}, ..., \lambda_{nN}\right)$. Without loss of generality the eigen values are assumed to be ordered in the decreasing order. Let

$$\delta_n^l = |\lambda_{nl} - \lambda_{n(l+1)}| \tag{24}$$

be the absolute value of the difference between two successive eigenvalues.

The codeword matrix error $\mathbf{E}_k \in \Gamma$ leading to a quick move from the diversity order n_R to $2 \times n_R$ is the one that verify

$$\delta_k^1 = \arg\min_{\mathbf{E}_n \in \Gamma} \delta_n^1. \tag{25}$$

Of course, this minimization could have many solutions but the outage probabilities of all possible solutions have the same behavior at the considered diversity order. In general, to move from the $l \times n_R$ diversity order to $(l+1) \times n_R$, one can select from the set of minimum determinant codewords the ones verifying

$$\delta_k^l = \arg\min_{\mathbf{E}_n \in \Gamma} \delta_n^l. \tag{26}$$

The proposed method is applied to find the suitable precoder for the 2×2 Golden Code and for the 4×4 Perfect Code.

IV. A 2×2 Golden Code and 4×4 Perfect Code Linear Precoding

A. Golden Code

The Golden Code is a (2×2) Space-Time code [9] which codewords entries are linear combinations of QAM information symbols s_1, s_2, s_3 and s_4. $\mathbf{W}$ has the following form

$$\mathbf{W} = \frac{1}{\sqrt{5}}\begin{bmatrix} \alpha(s_1 + \gamma s_2) & \alpha(s_3 + \gamma s_4) \\ i\bar{\alpha}(s_3 + \bar{\gamma}s_4) & \bar{\alpha}(s_1 + \bar{\gamma}s_2) \end{bmatrix}, \tag{27}$$

where $\gamma = \frac{1+\sqrt{5}}{2}$, $\bar{\gamma} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1+i-i\gamma$ and $\bar{\alpha} = 1+i-i\bar{\gamma}$. The vectorization of the received vector leads to :

$$\begin{aligned} \mathbf{Y}_{n_RT} &= \frac{1}{\sqrt{5}}\begin{bmatrix} \mathbf{H} & 0 \\ 0 & \mathbf{H} \end{bmatrix}\begin{bmatrix} \alpha & \alpha\gamma & 0 & 0 \\ 0 & 0 & i\bar{\alpha} & i\bar{\alpha}\bar{\gamma} \\ 0 & 0 & \alpha & \alpha\gamma \\ \bar{\alpha} & \bar{\alpha}\bar{\gamma} & 0 & 0 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \mathbf{B}_{n_RT} \\ &= \frac{1}{\sqrt{5}}\tilde{\mathbf{H}}\mathbf{MX} + \mathbf{B}_{n_RT}, \end{aligned} \tag{28}$$

where $\tilde{\mathbf{H}}$ is the duplicated channel matrix, $\mathbf{M}$ is the matrix of Golden Code and $\mathbf{X}$ is the vector of information symbols.
The Golden Code has the following properties:
- Full-diversity: equal to 4
- Full-rate : 2 symbols by channel use.
- Non-vanishing determinant for increasing spectral efficiencies equal to $1/5$.
- Optimal for the Diversity Multiplexing-gain trade-off.

In the following, we choose two error matrices having both the minimum determinant for the QPSK constellation. Their vectorial form is

$$E_1 = \frac{1}{\sqrt{5}}\frac{1}{\sqrt{2}}\mathbf{M}\begin{bmatrix} 0 & 0 & 0 & -2i \end{bmatrix}^T \tag{29}$$

and

$$E_2 = \frac{1}{\sqrt{5}}\frac{1}{\sqrt{2}}\mathbf{M}\begin{bmatrix} -2+2i & 0 & 0 & -2 \end{bmatrix}^T \tag{30}$$

The absolute value of the eigenvalues difference is $\delta_1 = 8.9443e-001$ for E_1 and is $\delta_2 = 5.7271$ for E_2 . The new criterion to select the best codeword difference among those having the minimum determinant is necessary. In figure 2, the outage probabilities of the system using the precoders based on the error matrices E_1 and E_2 are plotted. One can remark that the outage probability for the case with E_2 based precoding has a diversity of 2 for low SNRs (Precoder rank equal to 1) then the slope of the outage curve changes at $14dB$ and diversity becomes 4 for high SNRs. However, using for E_1 based precoding the diversity slope changing is done earlier.

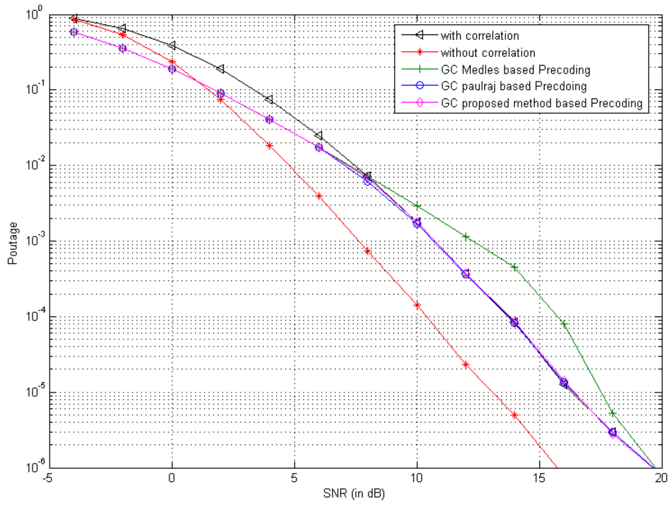

Figure 2: Outage Probability for the Golden Code

B. The 4×4 *Perfect Code*

A Codeword of the 4×4 Perfect Code can be written in this form[13]

$$\mathbf{W} = \frac{1}{\sqrt{15}} \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \\ \nu\psi(v_1) & \psi(v_2) & \psi(v_3) & \psi(v_4) \\ \nu\psi^2(v_1) & \nu\psi^2(v_2) & \psi^2(v_3) & \psi^2(v_4) \\ \nu\psi^3(v_1) & \nu\psi^3(v_2) & \nu\psi^3(v_3) & \psi^3(v_4) \end{bmatrix} \tag{31}$$

where $\nu = i$, $v_k = \sum_{l=1}^{4} S_{k,l}\phi^{l-1}$ and $\mathbf{S}_k = (S_{k1}, S_{k2}, S_{k3}, S_{k4})^T$, $k = 1, ..., 4$ are vectors composed of 16 information symbols. ϕ is defined as $\phi = \zeta_{15} + \zeta_{15}^{-1} = 2cos(\frac{2\pi}{15})$ where $\zeta_{15} = exp(\frac{i2\pi}{15})$ is the 15^{th} root of unity. The function ψ is

$$\psi : \zeta_{15} + \zeta_{15}^{-1} \longmapsto \zeta_{15}^2 + \zeta_{15}^{2\,-1} \tag{32}$$

As for the case of Golden Code, received codeword is represented in vectorial form.

V. Simulations and results

AoD (Angle of Departure) data provided with the 3GPP MIMO channel specifications [5] are used. It's assumed that each path is constituted of 20 sub-paths with a non frequency selective channel. A unit power for the path of interest with shadow fading equal to 1 are considered, and one can focus on the quasi-static case. Antennas have a linear arrangement and broadside orientation. The mean of AoD is 22.5 degrees and the angle spread is 35 degrees. The AoDs are generated with a Laplacian distribution [5]. The correlation matrix $\mathbf{R}$ is Toeplitz and can be written as

$$\mathbf{R} = \begin{bmatrix} 1 & |\rho| \\ |\rho| & 1 \end{bmatrix} \tag{33}$$

The maximum likelihood decoder is used for QPSK modulation (Sphere Decoder). Simulation results are obtained by averaging over 2000 independent Monte-Carlo trials where each burst consists of 100 data symbols. The correlation term is equal to $|\rho| = 0.9$. In figure 3 the bit error rate (BER) for

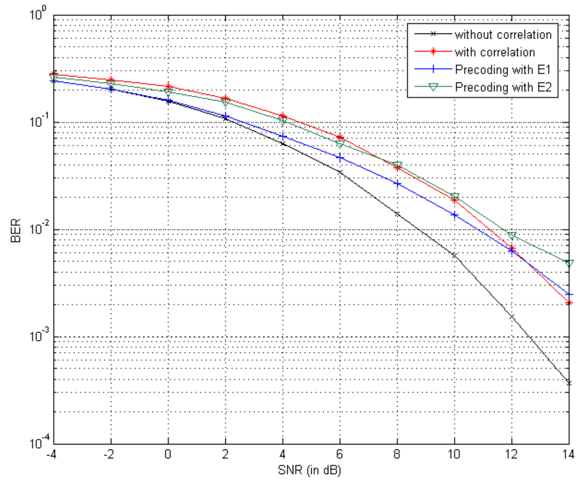

Figure 3: Golden Code precoding with two different codeword error matrices for QPSK

the precoders based on the two errors matrices E_1 and E_2 are plotted as a function of the SNR. Although both of them verify the minimum determinant, they don't provide the same performances. More over, at high SNRs the E_1 BER are lower than those of the system without precoding this is due to the lack of diversity (transmission on only one eigen mode).
During beamforming, only one row of Golden Code codeword is transmitted which makes a rank deficiency for the decoder. To face this problem, one can send only diagonal elements of codeword $\mathbf{W}$ instead of one row. This can reduce code rate since only two symbols are sent. To keep the same spectral efficiency one can adapt modulation and then the same rate is gotten.

Figure 4 shows the bit error rate (BER) for the i.i.d case, the cases with correlation and with precoding using a codeword error matrix verifying the proposed criterion. Our method and Paulraj one have the same results. This is expected since their outage probabilities are the same. These methods outperforms Medles selection which doesn't take into account the diversity influence. For 2×2 case the average design precoding is clearly simpler to obtain with a satisfying performance. However, this could not be the optimal solution for systems with higher number of antennas.

To prove this, one can focus on 4×4 Perfect Code. Similarly, we assume a channel in accordance with the 3GPP MIMO channel specifications [5]. The correlation matrix $\mathbf{R}$ is 4×4 Toeplitz matrix and can be written as

$$\mathbf{R} = \begin{bmatrix} 1 & |\rho_1| & |\rho_2| & |\rho_3| \\ |\rho_1| & 1 & |\rho_1| & |\rho_2| \\ |\rho_2| & |\rho_1| & 1 & |\rho_1| \\ |\rho_3| & |\rho_2| & |\rho_1| & 1 \end{bmatrix} \tag{34}$$

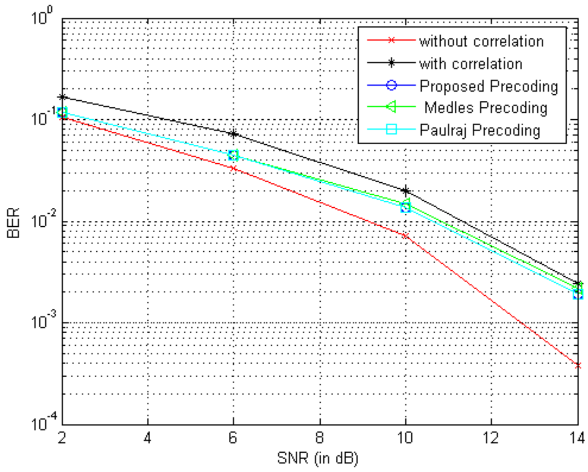

Figure 4: Golden Code in Correlated Channel with $|\rho| = 0.9$ for QPSK

For simulations, we fix correlation to the following values : $|\rho_1| = 0.9$, $|\rho_2| = 0.7$, $|\rho_3| = 0.5$. Figure 5 compares Paulraj precoding and proposed precoding. In the proposed precoding, to build precoder, an error matrix E is selected from the set of minimum determinant matrices and E verify the proposed criterion. Figure 5 shows clearly that the proposed precoding outperforms the Paulraj average design precoding. A gain of more than $1dB$ can be noticed for $SNR = 6dB$.

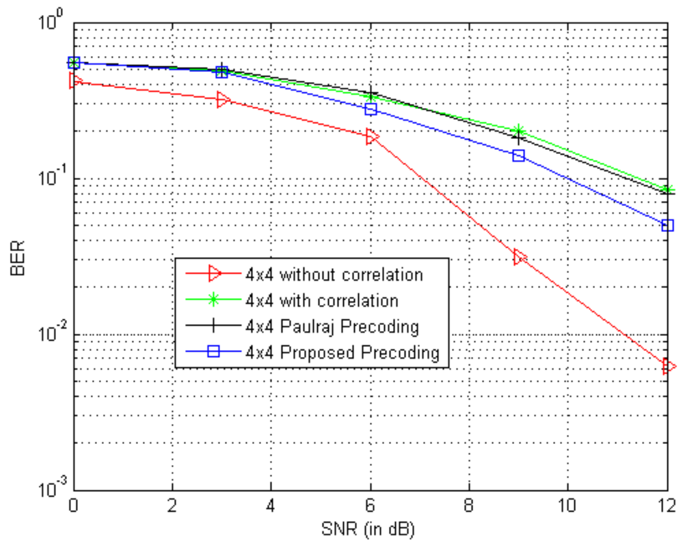

Figure 5: A 4×4 Perfect Code in Correlated Channel for QPSK

VI. Conclusions

Linear precoding for the non orthogonal STBC for MIMO correlated channel was addressed in this paper. It was shown how crucial is the choice of the code error matrix. A method for selecting this matrix was proposed. It is based on selecting the matrix having the minimum determinant and permitting to moving rapidly to higher diversity order as the SNR increases. This method was compared to Paulraj and Medles methods for 2x2 Golden code. The proposed method outperforms Medles ones but give same results as Pauraj for the Golden Code. Comparison was also held for 4×4 Perfect Code and results show clearly that proposed method is much better than the Paulraj average design.

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