An adaptive MIMO decoder

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***Abstract*—In existing MIMO systems, either optimal or sub-optimal decoders can be used according to the required performance. However, the optimal decoders give ML performance but have very high complexity and the sub-optimal decoders give low complexity but poor performance. Moreover for ML decoding, the variable decoding time at a fixed SNR for the different channel realizations and also the big gap in the complexity between low and high SNRs represent a critical point for practical implementation.**

We propose here an adaptive decoder that allows to switch between optimal and sub-optimal decoders according to the channel realization and the system specifications. This decoder offers an almost constant complexity while keeping good performance.

I. Introduction

In multiple-input multiple-output (MIMO) systems, different decoders have been proposed in the literature. These ones can be classified into two classes. In one hand, there are the optimal decoders like the sphere-decoder [1] and the Schnorr-Euchner algorithm [2]. These decoders have ML performances but exhibit a high complexity. On the other hand, there are the sub-optimal decoders that have low complexities but poor performances such as the ZF, MMSE and the DFE. In [3], a new sequential decoder for MIMO systems was introduced. This decoder is optimal, but using some parametrization, it offers a range of performances from ML to ZF-DFE with decreasing complexities.

For practical implementation, the decoding complexity represents a critical point, but also the big variation complexity between low and high SNRs is an additional problem because of the variable decoding time. In [3], the proposed decoders offer a complexity reduction of about 30% compared to the classical ones. So, we will resolve here the second problem of variable complexity. This will be enabled using the idea of adaptation.

We can find in the literature several schemes of adaptation. The most known ones are adapted modulation and adapted coding. They consist in adjusting the transmission parameters like the constellation size, the coding rate and the transmit power in order to maximize the transmission rate and to reach a target quality of service QoS (error probability). However, these adaptation methods only concern the transmitter side and the decoding remains unchanged.

We propose in this paper to apply adaptation in the decoding using the sequential algorithms. We define in the sequel selection criteria based on the channel quality and the system specifications. We show that the adaptive decoding scheme offers a constant complexity for all SNRs and good performance.

The paper is organized as follows: we first begin by introducing the system model, then we define the adaptive decoder and introduce the selection criteria. For each selection criterion, an implementation of the adaptive decoder is proposed and simulation results are given.

II. System model

We consider a MIMO system with M transmit and N receive antennas using spatial multiplexing. The received signal is given by

$$\boldsymbol{y^c} = \boldsymbol{H^c} \cdot \boldsymbol{s^c} + \boldsymbol{w^c} \qquad (1)$$

where $\boldsymbol{y^c} \in \mathbb{C}^N$, $\boldsymbol{H^c} \in \mathbb{C}^{N\times M}$ is the channel matrix with i.i.d components, $\boldsymbol{s^c} \in \mathbb{C}^M$ is the transmitted vector with components carved from a QAM constellation and $\boldsymbol{w^c} \in \mathbb{C}^N$ is the i.i.d complex additive white Gaussian noise vector with zero-mean and variance σ^2. Usually when $N = M$, we decompose the $N-$dimensional complex problem into $2N-$dimensional real problem as in [4] to get a lattice representation

$$\begin{aligned} y &= \begin{bmatrix} \Re\{\boldsymbol{H^c}\} & -\Im\{\boldsymbol{H^c}\} \\ \Im\{\boldsymbol{H^c}\} & \Re\{\boldsymbol{H^c}\} \end{bmatrix} \cdot \begin{bmatrix} \Re\{\boldsymbol{s^c}\} \\ \Im\{\boldsymbol{s^c}\} \end{bmatrix} + \begin{bmatrix} \Re\{\boldsymbol{w^c}\} \\ \Im\{\boldsymbol{w^c}\} \end{bmatrix} \\ &= \boldsymbol{H} \cdot \boldsymbol{s} + \boldsymbol{w} \end{aligned} \qquad (2)$$

In the coded case, using a linear space time block code [5] such as the TAST codes [6] and the perfect codes [7][8], the received signal is

$$\boldsymbol{Y^c_{N\times T}} = \boldsymbol{H^c_{N\times M}} \cdot \boldsymbol{X^c_{M\times T}} + \boldsymbol{W^c_{N\times T}} \qquad (3)$$

where $\boldsymbol{X^c_{M\times T}}$ is the codeword matrix. The equivalent channel matrix is now equal to the product of the channel matrix and the coding one [4]. And the lattice representation is obtained here by vectorization and separation of the real and imaginary parts of the received signal $\boldsymbol{Y^c_{N\times T}}$ [7].

As coded and uncoded MIMO system could be represented by equation (2), we will use for simplicity the spatial multiplexing scheme.

In the coherent case, where $\boldsymbol{H}$ is considered known at the receiver side, the ML detection problem consists in finding the information vector s minimizing

$$\hat{\boldsymbol{s}} = \arg \min_{\boldsymbol{s}^c \in QAM} \|\boldsymbol{y} - \boldsymbol{H} \cdot \boldsymbol{s}\|^2 \tag{4}$$

Then, this system can be resolved by using the lattice decoders like the sphere decoder or the sequential ones like the stack decoder. These decoders are based on tree-search algorithms. To apply them, we need first to expose the tree structure. A QR decomposition is then applied on the lattice generator matrix $\boldsymbol{H}$, the system (4) becomes

$$\hat{\boldsymbol{s}} = \arg \min_{\boldsymbol{s}^c \in QAM} \left\|\boldsymbol{Q}^{\dagger} \cdot \boldsymbol{y} - \boldsymbol{R} \cdot \boldsymbol{s}\right\|^2 \tag{5}$$

where $\boldsymbol{Q}$ is an orthogonal matrix and $\boldsymbol{R}$ an upper triangular one. The tree structure is given by $\boldsymbol{R}$. The branches of the tree define all the possible values that can be taken by the components of the vector $\boldsymbol{s}$. The node at level k in the tree is denoted by $s^{(k)} = (s_n, s_{n-1}, ..., s_k)$ and it is associated with the squared distance

$$f(s^{(k)}) = \sum_{i=k}^{n} f_i(s_i) \tag{6}$$

where $f_i(s_i) = \left|y_{1,i} - \sum_{j=i}^{n} r_{i,j} s_j\right|^2$, $\boldsymbol{y_1} = \boldsymbol{Q}^{\dagger} \cdot \boldsymbol{y}$. We call $f(s^{(k)})$ the cost of the node $s^{(k)}$. The tree search decoding consists then in exploring the tree nodes in order to find the path $(s_n, s_{n-1}, ..., s_1)$ with the least cost .

However, these decoders have a high complexity, especially for large constellations and high number of antennas where we have a big tree structure. Furthermore, the tree search takes too much time for low SNRs since the algorithm looks for all the possible points and then crosses more tree nodes to reach the optimal solution. This represents an additional problem since it leads to a too variable decoding time and so to a variable complexity. To overcome this problem, we propose in the sequel an adaptive decoding.

III. Adaptive MIMO decoder

A. Principle

Our adaptive decoding scheme will be based on the spherical bound stack decoder (SB-Stack) [3]. It is a sequential decoder that combines the sphere decoder search region [1] and the stack decoder search strategy [9]. The SB-Stack decoder offers a 30% complexity reduction compared to the sphere decoder while keeping the ML performance.

Moreover, it can be sub-optimal under some constraints like the stack decoder. In fact, by adding a parameter called the bias b in the cost function (6), we can write

$$f(s^{(k)}) = \sum_{i=k}^{n} f_i(s_i) - b \cdot k \quad , b \in \mathbb{R}^+ \tag{7}$$

The euclidean distance of $s^{(k)} = (s_n, s_{n-1}, ..., s_k)$ is lowered by $-b \cdot k$. Then, the algorithm will advantage the paths with the largest lengths in the tree since the smallest metrics are those of the deepest nodes. Under this constraint, the solution is not the ML one but depends on the value of b. However this allows the decoder to converge more rapidly.

As shown in Fig.1 for small values of b, near-ML performances are obtained since (7) is quite equivalent to the metric (6). The ML ones correspond to the case $b = 0$. And for large b the SB-Stack decoder becomes equivalent to the ZF-DFE. The complexities however are reduced as well as we increase b and for high SNRs we remark that the complexity remains almost the same for the different bias. We consider here the complexity as the total number of multiplications needed to decode one transmitted vector.

Recently, an adaptive decoder was proposed in [10] in the context of multi-user transmission. This one consists on switching from the RAKE to the LMMSE receivers according to the value of the SINR.

Generally, one can define an adaptive decoding scheme by selecting the appropriate decoder among a set of available decoders according to a selection criterion. Nevertheless, this means to set up at least two decoders at the receiver which increases the implementation complexity.

However in the MIMO case, the SB-Stack seems to be a good solution for adaptive schemes, as one has to change the value of the bias to switch from one decoder to another.

To define this adaptation scheme, we need to define selection criteria. We propose here criteria based on the channel quality, the system specifications and a combination of these laters.

B. Selection based on the channel state

This selection criterion was established based on an observation concerning the ML decoding. In fact, in some cases the search phase takes too much time without leading to the correct solution. These cases correspond to a "bad" channel realization. Until now to avoid this problem in practical implementation, the decoding is stopped after a fixed running time. But, such systems do not guarantee good performance.

So, the idea here is to detect these cases before decoding and replace the ML decoder by the sub-optimal one to avoid the high decoding delay and therefore the high complexity. This can be achieved by measuring the channel state. In this sense, some indicators can be deduced from information theory. The most reliable one is the channel capacity. This

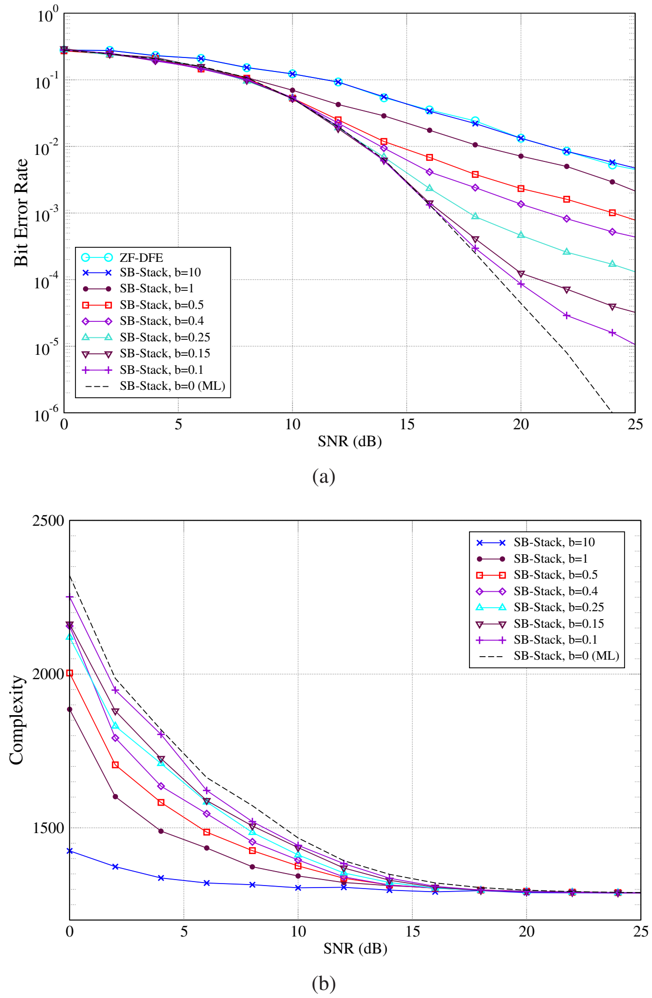


Figure 1. Performance and complexity of the SB-Stack decoder for 4×4 MIMO system with spatial multiplexing using a $16-QAM$ constellation for different values of the bias b

measure determines the amount of information that can be transmitted over the channel without error. By assuming an isotropic transmit signal, we define the instantaneous capacity (function of $\boldsymbol{H}$) as [11]

$$C(\boldsymbol{H}) = \log_2 \det \left(\boldsymbol{I}_N + \frac{\rho}{M} \boldsymbol{H}\boldsymbol{H}^t \right)$$

where ρ is the signal to noise ratio (SNR) per receive antenna. As we consider block fading channels, the coding performance is driven by the outage probability. This measure is defined as the probability that the capacity falls below the effective data rate R (given in bits per channel use)

$$P_{\text{out}}(R) = \Pr\left\{ C(\boldsymbol{H}) < R \right\}$$

- if $C(\boldsymbol{H}) < R$, the channel is in outage, *i.e*, the channel quality is so bad that decoding the transmitted codeword with a low error probability is impossible even using the ML decoder. In this case, it is then more judicious to use sub-optimal decoders.
- if $C(\boldsymbol{H}) \geq R$, the channel is not in outage, *i.e*, it is possible to decode the transmitted data with a low error probability. In this case, we use a ML decoder.

At each transmission frame, the adaptive decoding consists then on computing the instantaneous capacity $C(\boldsymbol{H})$. If the channel is not in outage, the ML decoder (the SB-Stack decoder with $b = 0$) is used, and if the channel is in outage the sub-optimal decoder (the SB-Stack decoder with $b = 10$ which corresponds to the ZF-DFE) is used to decrease the complexity.

In Fig.2, we plot the performances of the ML, the ZF-DFE and this adaptive decoder (Selection/Pout) as a function of SNR for a 4×4 MIMO system using 16-QAM constellation and spatial multiplexing.

We observe that the adaptive decoder offers performances at about 3dB from ML. However, we can see that its complexity is almost constant for all SNRs. The problem of variable complexity is so resolved. Besides, as we can see in Fig.3, this selection allows to have at fixed SNR an almost constant decoding time for the different channel realizations compared to the ML case.

C. Selection based on the system specifications

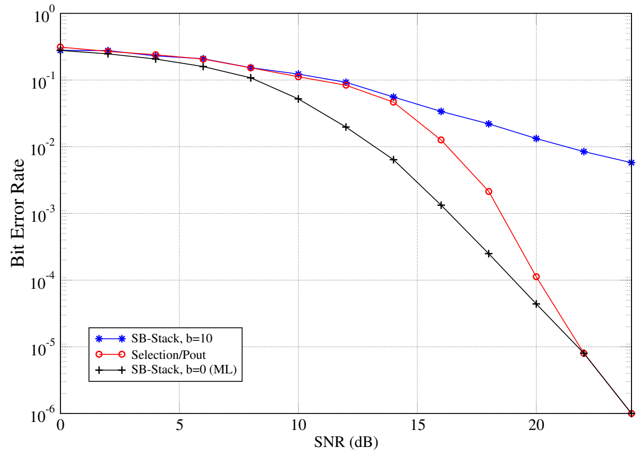
A natural selection criterion is the system specification as in existing adaptive schemes in the literature. The objective of using this second criterion is to guarantee a target QoS. In fact, unlike the first criterion when the error probability is not fixed preliminary, in this case we establish a table of target error probabilities that we want to reach at each range of SNR. The receiver will then select the decoder to run that allows to achieve this performance.

We give in the sequel an example of adaptive decoding using this criterion. We consider the former 4×4 system and we consider a system specification that imposes at maximum 2dB loss from ML. According to the performances obtained in Fig.1-a, we select the appropriate bias to obtain the designated specification.

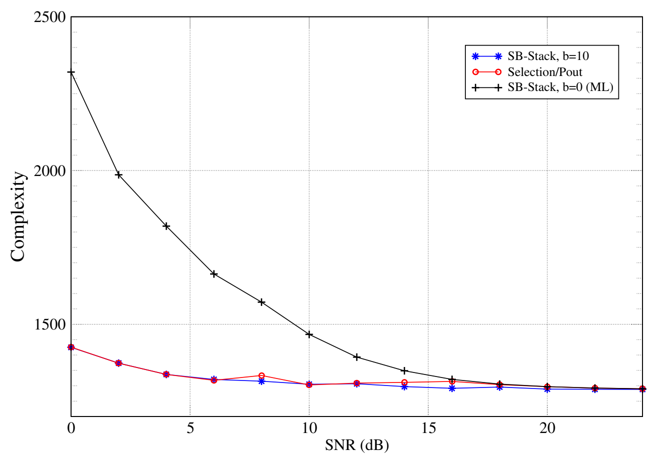
In Fig.4, we plot the error rates and complexities of this selection method denoted by Selection/Spec. We note that this later offers performances better than the adaptive decoder using the first criterion. This is completely predictable as the performance is preliminary fixed. Furthermore, the complexity is slightly higher as more complex sub-optimal decoders are used ($b \leq 10$) but it remains almost constant.

D. Combined selection

The first selection criterion is based on the instantaneous capacity and so makes an instantaneous selection of the corresponding decoder (ML or ZF-DFE). The second selection criterion is based on the system specifications and



(a)



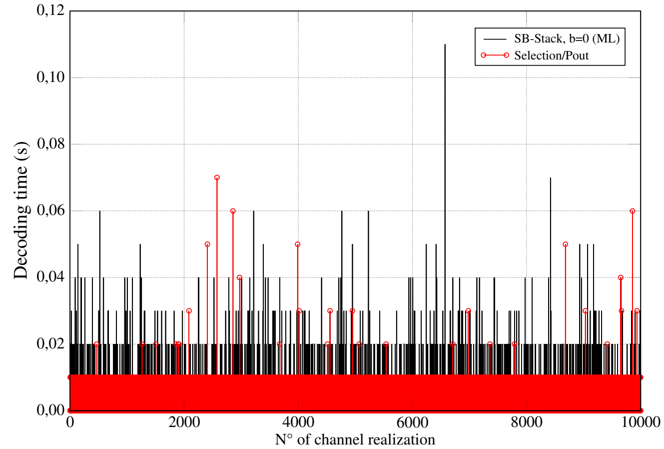
(b)

Figure 2. Performance and complexity of the adaptive decoder based on the channel state, MIMO 4×4, 16-QAM constellation

so selects the decoder for each range of SNR.

We propose here a third criterion combining these two ones. The idea is to define in a first time several SNR ranges using the second criterion, and in a second time for each range of SNR, we select instantaneously the decoder based on the first criterion (the instantaneous capacity). So, in each SNR range, if the channel is in outage the stack decoder with the fixed bias is used, and if the channel is not in outage the stack decoder with $b = 0$ (ML) is enabled.

To evaluate this adaptive method, we consider the same MIMO system as in the two former cases. The performance corresponding to the third criterion is denoted by 'Combined selection'. We remark that we obtain better performances than the two last selection methods. This is foreseeable as compared to the first criterion, better sub-optimal decoders are used when we are in outage. And compared to the second method, the "bad" channel realizations are well treated thanks



(a)

Figure 3. Decoding time for different channel realizations (10000 iterations) at SNR=12dB, MIMO 4×4, 16-QAM constellation

to the computation of the outage probability. We remark also that the complexity is almost always the same as the adaptive decoder using the system specifications.

Besides, for the second criterion we switch brutally from one decoder to another in order to get the specified error rate in each range of SNR. The obtained curve of performance is then a piecewise curve. However in the combined selection, we obtain a smoother curve since we consider the instantaneous channel state and so adaptation is made instantaneously.

IV. Conclusion

We proposed in this paper an adaptive MIMO decoder based on the channel quality and the system specifications. The main feature of this one is to have a constant complexity while keeping a good performance and so to resolve the problem of variable complexity of the classical decoders.

But, the implementation of the adaptive decoding requires several decoders at the receiver side. So, the SB-Stack one represents a good solution as it offers a range of performances from ZF-DFE to ML with respective complexities. The adaptation is then performed by only adjusting the value of the bias. However, this does not restrict to define adaptive decoding using other optimal and sub-optimal decoders. The price to pay is an additional implementation complexity.

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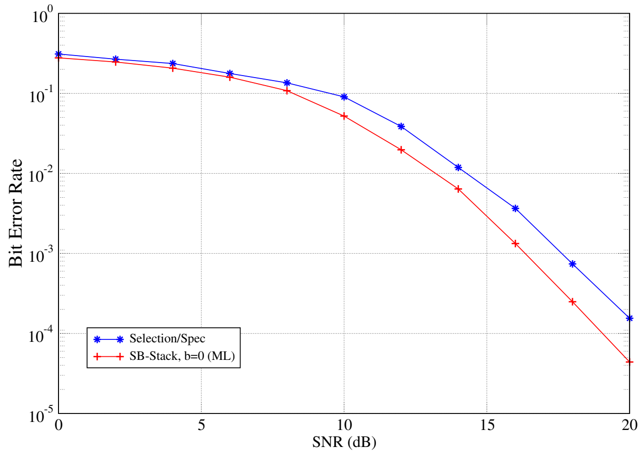

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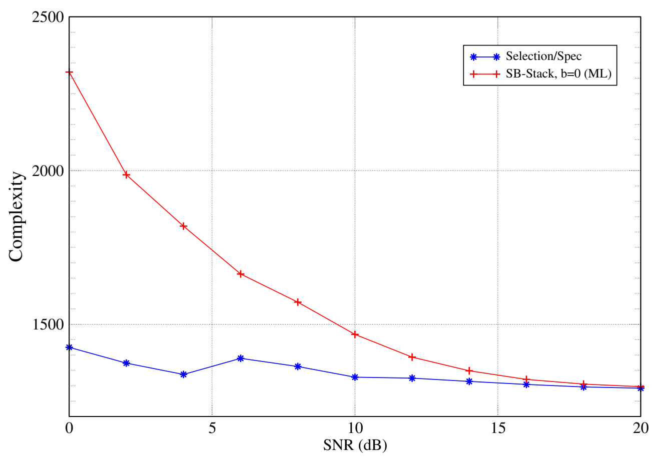

(b)

Figure 4. Performance and complexity of the adaptive decoder based on the system specifications, MIMO 4×4, 16-QAM constellation

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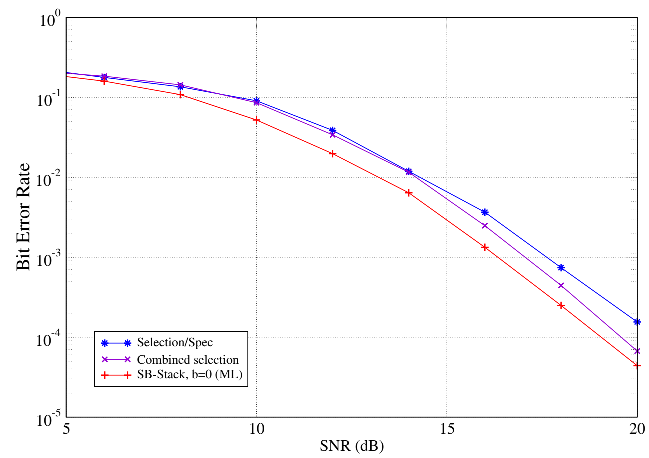

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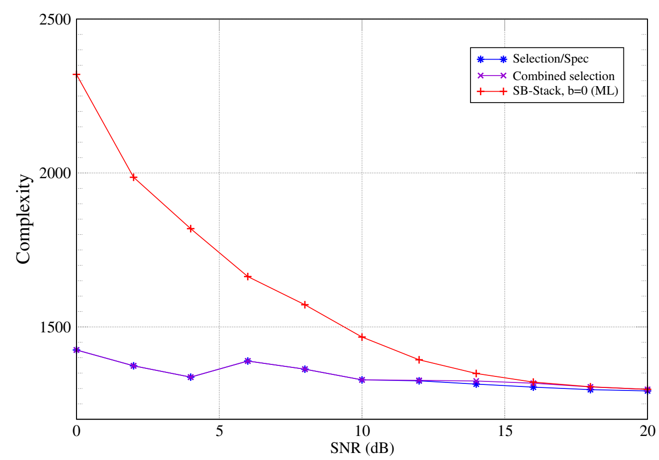

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Figure 5. Performance and complexity of the adaptive decoder based on the combined selection, MIMO 4×4, 16-QAM constellation