On the Golden Code Performance for MIMO-HSDPA System

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***Abstract*— In this paper, we propose a new MIMO-HSDPA transmission scheme with two transmit and two receive antennas, using an optimal Space-Time block code: the Golden code [1]. This code has a full rate, a full diversity, achieves the Diversity-Multiplexing gain tradeoff and preserves the mutual information. The proposed scheme is compared to the RCMPD [2] a scheme which has also a full rate and a full diversity. The major disadvantage of the later is the generation of the Multiple Access Interference (MAI) even on a flat fading channel. Simulation results show that the proposed scheme has better performances specially for high spectral efficiency. The RCMPD performance are hardly affected by the MAI.**

I. Introduction

The High Speed Downlink Packet Access (HSDPA) is specidied in 3GPP Release'5 in order to increase the downlink throughput [3] and to achieve a data rate of about 14 Mbps. This packet-swiched system is completely compatible with the Universal Mobile Telecommunication System (UMTS) and is based on adaptive transmission related to the channel quality. This principle is based on two concepts: multiuser diversity and link adaptation. This diversity is obtained by allocating the cell resource to the user having the best channel condition by a scheduler located at the Node B. The link adaptation consists on assigning the Modulation and Coding Scheme (MCS) with the higher througput according to the Channel Quality Indicator (CQI). The QPSK and 16 QAM modulations could be used and multi-code transmission is permitted. The user selection and the link adaptation are done every 2 ms which corresponds to the Time Transmission Interval (TTI). This shortened frame duration permits to track the fast fading. Retransmission protocols (Hybrid ARQ) consisting in Chase Combining or Incremental Redundancy are also introduced.

The achieved rates in this standard could be increased by introducing multiple antennas techniques. It is well known that at high signal to noise ratio the multiple-input multiple-output (MIMO) system capacity increases linearly with the minimum number of antennas at the transmitter and the receiver sides. Moreover, Space-Time Codes (STC) can be used to provide a temporal and spatial multiplexing and hence to improve the MIMO scheme performance. The best known and used Space-Time Block codes (STBCs) are the Alamouti code [4] and the VBLAST scheme [5] (also known as spatial multiplexing). Unfortunatly, both STBCs are not optimal, the first one in terms of multiplexing rate and the second one in terms of diversity.

In this work, we are interested to optimal STBCs which have full-rate, full-diversity, achieve Diversity-Multiplexing gain tradeoff and preserve mutual information [6]. A new family of optimal STBCs, called Perfect codes is presented in [7][8]. The Golden code [1] is the best Perfect code for two transmit and two or more receive antennas.

The application of the MIMO techniques to HSDPA system may provide higher throughputs. Several transmission schemes are proposed for this purpose [2]. We propose in this paper a new MIMO-HSDPA transmission scheme based on the Golden code. To make fair comparaison with the existing schemes, we have chosen the ones having full rate and full diversity. The only candidate satisfying both previous properties is the RCMPD scheme [2].

This paper is organized as follows. In section II the system model is defined. The Golden Code construction and the Diversity-Multiplexing gain tradeoff are described in section III. The new MIMO-HSDPA scheme and the RCMPD are presented in section IV. Simulation results and discussions are provided in section V.

II. System Model

We consider here the coherent case where the receiver perfectly knows channel coefficients. Let M and N be the respective numbers of transmit and receive antennas, and T the temporal codelength. The received signal is :

$$\mathbf{Y}_{N\times T} = \mathbf{H}_{N\times M} \cdot \mathbf{X}_{M\times T} + \mathbf{W}_{N\times T} \tag{1}$$

where $\mathbf{X}$ is the transmitted codeword taken from STBC, $\mathbf{H}$ is the channel matrix and $\mathbf{W}$ is the noise matrix. Indices correspond to the respective matrix dimensions. The $M.T$ information symbols are carved from QAM constellations and mapped onto the code word $\mathbf{X}$ which belongs to the codebook $\mathcal{C}$. Entries of $\mathbf{W}$ are assumed to be i.i.d centered complex Gaussian random variables with variance N_0.

In [9], Tarokh *et al.* established Space-Time code design criterions in order to improve diversity and coding gain. They are based on the asymptotic pairwise error probability. We remind here these two criterions and for this, let us define $\mathbf{A} = (\mathbf{X} - \mathbf{T})(\mathbf{X} - \mathbf{T})^H$, where $\mathbf{X}$ and $\mathbf{T}$ are two distinct

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codewords in $\mathcal{C}$. The diversity order of a MIMO scheme is the signal to noise ratio exponent, which also corresponds to the asymptotic slope of the error rate curve.

Criterion 1: **Rank criterion:** To achieve maximal diversity order $N \cdot M$, matrix $\mathbf{A}$ must be of maximum rank for all codewords $\mathbf{X}$ and $\mathbf{T}$.

For the same order of diversity, the coding gain measures the gain between the uncoded and coded schemes.

Criterion 2: **Determinant criterion:** To maximize the coding gain, the minimum determinant of $\mathbf{A}$ must be maximized for all codewords $\mathbf{X}$ and $\mathbf{T}$. That is

$$\delta(\mathcal{C}) = \max\left(\min_{\mathbf{X},\mathbf{T}\in\mathcal{C},\mathbf{X}\neq\mathbf{T}}\left(\det\left(\mathbf{A}\right)\right)\right)$$

III. Golden Code

A. Code description

The Golden code presented in [1], was designed for a MIMO system with $M = 2$ and $N \geq 2$. A very interesting and powerful mathematical tool was used in the code construction: cyclic division algebras. A division algebra naturally yields to a structured set of invertible matrices that can be used to construct Linear Dispersion STB codes [10].

The Golden code can be considered as a subset of the cyclic division algebra $(\mathbb{Q}(i,\sqrt{5}), i)$ with center $\mathbb{Q}(i)$. It has been named in such a way because of the key role played by the Golden number $\frac{1+\sqrt{5}}{2}$ in the construction. The codeword matrix is :

$$\mathbf{X} = \frac{\mathbf{1}}{\sqrt{\mathbf{5}}}\begin{bmatrix} \alpha(s_1+\theta s_2) & \alpha(s_3+\theta s_4) \\ i\bar{\alpha}(s_3+\bar{\theta}s_4) & \bar{\alpha}(s_1+\bar{\theta}s_2)\end{bmatrix}$$

where $\theta = \frac{1+\sqrt{5}}{2}$, $\bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1+i-i\theta$, $\bar{\alpha} = 1+i-i\bar{\theta}$ and s_i are QAM information symbols, $i = 1\cdots 4$. Its minimum determinant is a constant integer :

$$\delta_{min} = \min_{\mathbf{X}\neq 0}\left(\left|\det\left(\mathbf{X}\right)\right|^2\right) = \frac{1}{5}$$

The Golden code is optimal in several senses. It has :

- Full rate (2 symbols per channel use)
- Full diversity ($d = 4$)
- Non-vanishing determinant, it is clear that δ_{min} is independent of the constellation size.
- a preserved mutual information (unitarity of the vectorized codeword matrix).

A simple vectorization of equation 1, leads to a lattice representation of the system. Thus, we can use lattice decoding, such as sphere decoder or Schnorr-Euchnerr decoder [11].

In [7], an infinite STBC family for 2 transmit antennas is constructed by replacing the Golden number by $\frac{1+\sqrt{p}}{2}$, where p is a prime number such that $p \equiv 5 \mathrm{mod}\ 8$. Its minimum determinant is $\delta_{min} = \frac{1}{p}$. The Golden code is the best code in this family, and is the best known STBC for MIMO schemes with 2 transmit antennas.

B. Diversity-Multiplexing Gain Tradeoff

The diversity order measures the reliability of a communication system on a fading channel, and the multiplexing gain is related to the data rate that can be transmitted. A MIMO system has a maximal diversity gain equal to $M \cdot N$ for a fixed rate and a maximal multiplexing gain equal to $\min\{M, N\}$. Unfortunately, the maximisation of the diversity gain implies the minimisation of multiplexing gain. Zheng and Tse in [6] have introduced a fundamental tradeoff between the two quantities.

Let R be the transmission rate and r be the normalized rate given by $R = r\log(SNR)$ (the multiplexing gain corresponds to an increasing rate function of the signal to noise ratio denoted by SNR), then the multiplexing gain and the diversity gain are :

$$r \triangleq \lim_{\text{SNR}\to\infty}\frac{R(SNR)}{\log(SNR)}, \qquad d(r) \triangleq -\lim_{\text{SNR}\to\infty}\frac{\log(P_e)}{\log(SNR)}$$

where P_e denotes codeword error probability.

Elia *et al.* proved in [12], that a MIMO system employing linear Space-Time coding, having a full rate and non-vanishing determinant, achieves the optimal diversity-multiplexing gain tradeoff.

In figure 1, the Diversity-Multiplexing Gain of Alamouti, VBLAST and Golden code are plotted. We can see that :

- for the Alamouti code, the maximum multiplexing gain is equal to 1, and the maximum diversity gain is equal to 4
- for the VBLAST scheme, the maximum multiplexing gain is equal to 2, and the maximum diversity gain is equal to 2
- for the Golden code, the maximum multiplexing gain is equal to 2, and the maximum diversity gain is equal to 4

The Alamouti code and the VBLAST scheme are not optimal as they do not achieve the tradeoff which is also the tradeoff of the Golden code.

IV. MIMO-HSDPA schemes

The open loop transmission schemes for MIMO-HSDPA use as STBC the Alamouti code or the VBLAST scheme [2]. The considered scheme that has to be compared to the proposed one is named Rate Control Multipath Diversity. Indeed it seems to have the same properties as the Golden code based one. In the following subsection the two schemes are presented.

A. Proposed scheme

The MIMO-HSDPA transmission scheme using Golden code is shown in figure 2.

The transmitted bits are split into two streams. In each one, one transport block arrives to the turbo-coding unit, every TTI

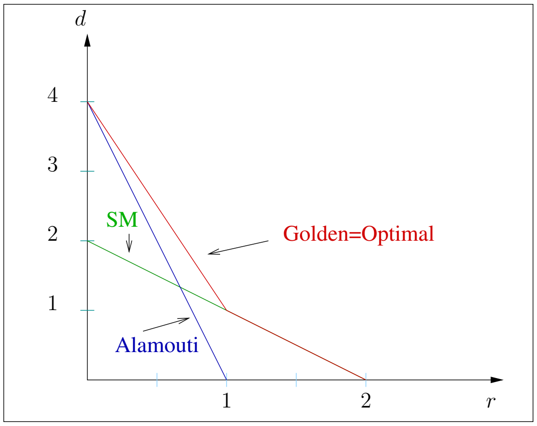

Fig. 1. Diversity-Multiplexing Gain for some MIMO schemes with $M = 2$, $N = 2$.

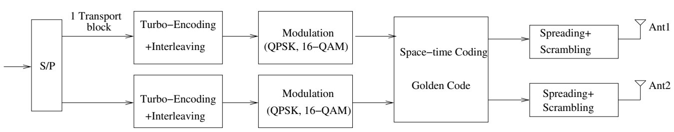

Fig. 2. The Golden code based MIMO-HSDPA transmission scheme.

(which is equal to 2ms). The rate of the channel coding is $\frac{1}{3}$. Then coded bits are punctured or repeated depending on the rate matching constrains [13]. Then the resulting block is segmented into K sub-blocks, where K is the number of channelization codes allocated to the current user. Sub-block size is equal to 960 bits for QPSK modulation, and 1920 bits for 16-QAM modulation. The bits are then interleaved, to produce uncorrelated output bits. The resulting blocks are mapped to QPSK or 16-QAM complex symbols and then coded by the Golden code. The obtained codewords blocks (2×2 matrix) are spread by the channelization codes. The two parallel branches use the same codes. All these codes are real OVSF, sometimes called Walsh codes, with a fixed spreading factor SF=16 [14]. Finally, the chip sequences are summed then scrambled by a complex-valued scrambler.

The modulation and coding scheme MCS are chosen according to the CQI. This measure is taken by the user equipment and sent over a signalling channel. If its radio conditions are favourable which means a high CQI, all the resources (up to 15 channelization codes) may be allocated to this user. The relation between the CQI and the MCS is described in [3]. The proposed scheme has two streams which are separately coded and each one can use a MCS different from the other one.

At the receiver side, after despreading and descambling, the Golden code word has to be decoded to obtain the modulated symbols. For that, the Sphere-Decoder (SD) [11], a ML decoder, is used. In fact, the use of such a decoder is made possible due to the Golden code lattice representation obtained by vectorization and separation of real and imaginary parts of matrix codeword. The SD searches the transmitted point in a sphere centered in the received point, by looking at all the points inside the sphere. The sphere radius is calculated as a function of the noise variance, to optimize and accelerate the search. For a MIMO system with $M = N = 2$, it's interesting to use the SD as it leads to ML-performance with a reasonable complexity.

B. RCMPD scheme

The Rate-Control Multi-Paths Diversity (RCMPD) MIMO-HSDPA scheme presented in figure 3 is based on multi-stream transmission. Each stream is transmitted at least through two antennas. This means that this scheme has a full diversity.

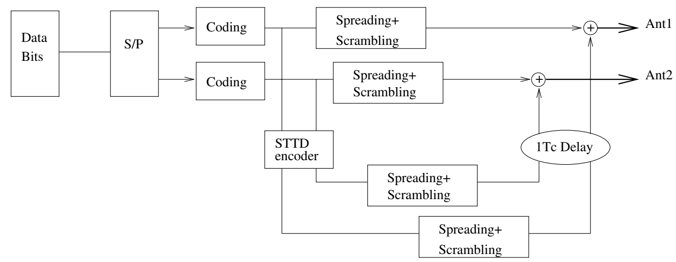

Fig. 3. RCMPD MIMO-HSDPA transmission scheme.

Data bits are first demultiplexed into two independent streams. Each one is coded, modulated, spread, scrambled and then sent from one antenna. After the modulation, another copy of the streams are encoded with a Space Time Transmission Diversity encoder (STTD) [2] which is equivalent to the Alamouti code. Indeed, by denotting (s_1, s_2) the input of this encoder, the output is $(-s_2^*, s_1^*)$. The encoder outputs are spread and scrambled by the same sequence as the direct transmitted streams. In order to be able to separate the two STTD encoder streams to the direct ones, a delay of one chip duration is introduced before the summation of each direct stream and encoded one. This operation is equivalent to the use of new spreading codes which are not perfectly orthogonal. Since the delay is short compared to the symbol one this scheme could be considered as a full rate one. Of course, the major inconvenient of this scheme is the generation of the inter-code interference even on a non frequency selective channel.

At the receiver side, at least two antennas are needed. The received signal is despread to obtain a linear combination of the direct transmitted streams. In parallel, the signal is delayed by one chip then despread to obtain a linear combination of the STTD encoder outputs. The two obtained streams are decoded by the simple Alamouti ML decoding.

V. Simulations Results

A. Uncoded scheme

In a first time, we present the Golden code, the Alamouti code and the VBLAST uncoded performance for a non-selective channel. The transmission scheme consists of QAM-modulation followed by Space-Time coding. There is no channel coding.

In figure 4 the symbol error rates of the considered STBCs as a function of $\frac{E_b}{N_0}$ are compared. The common spectral efficiency is 8 bits/channel use. We recall that the asymptotic slope of the error rate curve gives the diversity order of the MIMO scheme. The respective maximum diversity order of Golden code, Alamouti code and VBLAST scheme are respectively 4, 4 and 2 as given by the Diversity-Multiplexing Gain tradeoff. For the Alamouti code the use of 256-QAM constellation instead of 16-QAM constellation for the Golden code is needed in order to get the same spectral efficiency. Moreover, it's hard to obtain higher bit rates with the Alamouti code. Also, we can remark a performance loss about 5dB of the Alamouti code compared to the Golden code.

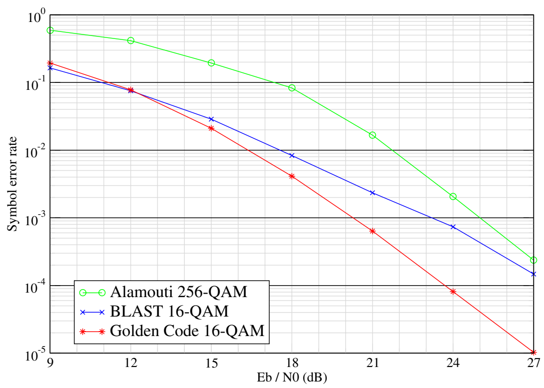

Fig. 4. Performances of Alamouti, VBLAST and Golden code, $M = N = 2$.

B. Coded scheme

MIMO-HSDPA schemes including channel coding are now considered. For the HSDPA system, there is up to 30 MCSs depending on the user equipment capability [3]. In this work and for convenience, only some of them are chosen and they are described in table I. The simulated MIMO channel is generated respecting the 3GPP specifications [15]. The channel is assumed to be constant on one TTI. The number of receiver antennas M is equal to 2.

TABLE I
MODULATION AND CODING SCHEME TABLE.

MCS	Transport block size	Coding Rate	Spreading code number	Modulation
8	792	0.41	2	QPSK
10	1262	0.43	3	QPSK
12	1742	0.6	3	QPSK
14	2583	0.67	4	QPSK
15	3319	0.69	5	QPSK
16	3565	0.37	5	16 QAM

The performance is first evaluated by considering a flat fading channel. In figure 5, the frame error rate of the two schemes are compared for the MCS15. The choice of this MCS is not restrictive and argued by the use of the maximum code number 5 for the most robust modulation QPSK. This leads to observe clearly the effect of the multicode interference generated by the RCMPD. There is a degradation of about $1dB$ for a $FER = 3.10^{-2}$.

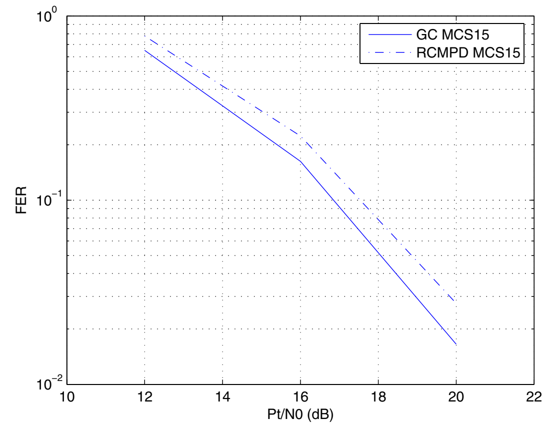

Fig. 5. Proposed and RCMPD scheme performance over flat fading channel.

The interference effects could be amplified when the channel is frequency selective. Indeed, the interpath interference arises and is added to the RCMPD one. For such channel, the two schemes use a chip level space-time LMMSE equalizer successed by their ML detectors. In this case, the detectors consider the channel and the equalizer convolution as a global non frequency selective channel. The performance for different MCSs and a Vehicular-A channel between each transmitter and receiver antenna are shown in figure 6. For low MCSs the two schemes have the same performance. Indeed, their diversity orders are equal, the number of spreading codes used is low and the interference is drowned in the noise. For MCSs 14 and 16, the GC based scheme outperforms the RCMPD one by about 2dB at $FER = 10^{-1}$. For the further MCS, the number of its spreading codes and the coding rate increases which amplify the interference effect. For the later, even if its coding rate is low, the 16 QAM modulation sensitivity to interference affects hardly the performance. The gap between the two HSDPA MIMO scheme performance might increase for higher MCSs.

VI. CONCLUSIONS

A new HSDPA-MIMO transmission scheme using the Golden code has been proposed. The Golden code is an optimal Space-Time Block Code, has full rate, full diversity, achieves the Diversity-Multiplexing tradeoff and preserves the mutual information. Simulation results show that the Golden code has the best performance compared to the Alamouti code and to the VBLAST scheme for uncoded transmission scheme. Moreover, for the MIMO-HSDPA transmission, the new scheme outperforms the RCMPD one for non-selective and selective channels.

It could be interesting to compare the performance of the others MIMO-HSDPA schemes described in [2] to the proposed one. This work has to be done for the same spectral efficiency. When two schemes have different space-time code

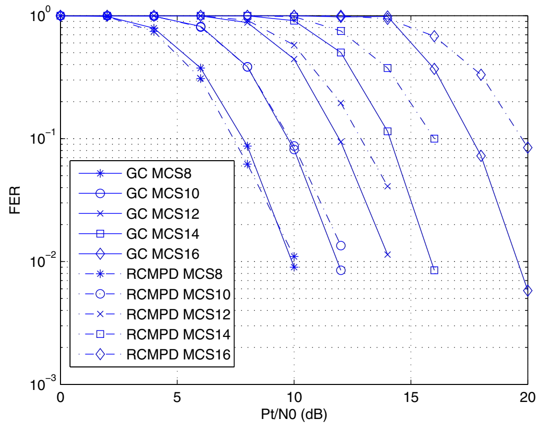

Fig. 6. Proposed and RCMPD scheme performance over frequency selective channel.

spectral efficiency, it is possible to obtain the same one by using different MCSs.

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