Bounded Delay-Tolerant Space Time Block Codes for Asynchronous Cooperative Networks

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***Abstract*—When distributed cooperative nodes are communicating with a destination, the received signal can be asynchronous due to the propagation or processing delays. This can destroy the space time block code properties designed initially for synchronous case. In this paper, a new construction method of bounded delay tolerant codes is presented. These new codes preserve the full diversity with optimal rates if the relative delays are in a designed delay tolerance interval. The general design method is based on the concatenation and permutation of optimal synchronous space time block codes and works for an arbitrary number of transmitting and receiving antennas. Examples of bounded delay tolerant codes based on the Alamouti code, the Golden code and Threaded Algebraic Space-Time (TAST) codes are given. Theoretical proofs are used to show that the new codes respect the design criteria. Simulation results manifest better error rate performance of the new codes compared to other known delay tolerant codes.**

***Index Terms*—Delay tolerant codes, asynchronous cooperative networks, distributed space-time coding, rank and determinant criteria, cooperative diversity.**

I. Introduction

COOPERATION between the nodes of a wireless network is a promising technique to increase the reliability and the transmission rate of data over the wireless channels. This can be an alternative technique of multiple-antenna (MIMO) systems to provide spatial diversity when network nodes cannot have more than one antenna due to size, cost, or hardware limitations. Space Time Codes (STCs) were proposed for MIMO systems and distributed versions of STCs were also designed for cooperative communications. These codes follow the well-known rank and determinant criteria [1] when the nodes of the network are synchronous [2]–[5]. However, unlike the multiple-antenna (MIMO) systems where the antennas are collocated at the same device, the antennas in cooperative systems are spatially distributed on different nodes. This new configuration can result in an asynchronism due to the difference in local oscillators and/or the different processing and propagation delays. The lack of perfect synchronization among the cooperative transmitting nodes destroys the required Space-Time Code signal structure leading to the reduction of the achievable diversity and thus deteriorates the code performance. Therefore, the synchronous STC designed for MIMO systems are no longer valid for asynchronous cooperative communications.

Many recent works have proposed solutions to preserve the diversity order when the relays have arbitrary relative delays in transmitting their symbols [6]–[9]; for instance, the Orthogonal Frequency Division Multiplexing (OFDM) technique, or the design of a Space-Time Block Code (STBC) that is delay tolerant i.e. the code matrix remains full rank even in the presence of a delay between the relays. In [7], to reduce the impact of synchronization errors, the authors proposed to use Space-Time Codes designed for frequency-selective channels to combat these errors; in particular, Time-Reverse Space-Time Code (TR-STC) and Space-Time OFDM (ST-OFDM) were considered. The authors of [8] designed a distributive Space-Frequency code based on OFDM for frequency selective fading channels. Cyclic prefix is used at the relays to combat the timing errors and the delay of multipath. However, these solutions are not suitable for non OFDM systems and they cause a rate loss that results from adding cyclic prefix.

On the other hand, several codes preserving these properties in the case of lack of synchronization have been proposed and called "delay tolerant codes". In [10], the author showed that codes obtained form generalization of the construction in [11] preserve the diversity gain despite the timing offset among the relay nodes. He also showed that certain binary STBC derived from the stacking construction [12] are delay tolerant.

In [13], the authors built on the framework provided by [14] and [15] to design a new class of STBC that shares the advantages of the Threaded Algebraic Space-Time (TAST) codes and are also delay tolerant. Their proposed codes are referred to as "Distributed TAST codes" with length growing exponentially with the number of relays. Nevertheless, to achieve a full diversity for any delay, the transmission rate is reduced by the repetition of some symbols. This solution has been applied to the optimal synchronous codes such as the Alamouti code [16] and the Golden code [17]. Another delay-tolerant extension of the TAST is presented in [18]. In difference with [13], the code construction was restricted to square matrices (the temporal span of the code equals the number of transmit antennas) and has used a full-diversity single input single output (SISO) code to fill all the threads in the space-time codeword matrix.

In [19], a new 2×2 delay tolerant code is proposed based on

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modification of the Golden code by application of convenient unitary matrices. The idea is to combine differently all the symbols to send, for each antenna and each transmission. However, to successfully decode these codes, we should have a sufficient number of antennas at the receiver which means that single antenna nodes cannot use these codes. In [20], we introduced a new coding scheme for two asynchronous transmitters based on the outage probability derivations in [21]. But, this scheme is not practical because the diversity order becomes equal to 1 when the transmitters are synchronous.

Previous works looked for a relevant solution for any delay value and this usually results in a loss of rate and an increasing complexity. However, in practical wireless systems the delays are generally bounded by a maximal delay as in [22], which motivates our work. In this paper, we introduce a new design construction method based on optimal synchronous codes, to build delay tolerant codes for an arbitrary number of cooperative nodes and for certain delay profiles. These new codes will be referred to as "bounded delay tolerant STBC". Moreover, examples of these codes based on Alamouti, Golden and TAST codes are given. The new codes are shown to respect the rank criterion and determinant criteria and to outperform other known delay tolerant codes.

The contributions of this work can be summarized as follows:

- The design of new family of distributed delay tolerant STBCs based on optimal synchronous codes for cooperative communications. The new codes ensure optimal performances when the cooperative nodes are synchronous and a full diversity and optimal rate for a certain set of delay profiles that change with the code length.
- The design method is simple to apply and it consists of changing the order of the symbols to send by concatenating and permutating several code matrices.
- The method is also flexible with respect to the number of transmit and receive antennas, the signaling constellation and the transmission rate.
- The overall rate of the code is optimized by increasing the code length and thus less guard intervals are needed between the codewords because the length of the codewords are bigger.

The paper is organized as follows. In Section II, the system model is described. The delay tolerance of some known STBC and existing solutions are discussed in Section III. In Section IV, the general construction method of bounded delay tolerant codes is explained and the advantages of this method are discussed. Construction examples based on the Alamouti code, Golden code and TAST codes are designed in Sections V and VI with theoretical proofs and simulation results. In Section VII, we give conclusions.

II. System Model

We consider a wireless system with M transmitters $T_1, T_2, \ldots, T_M$ having one antenna each, and a destination D with N_r antennas. The network model is shown in Figure 1.

Due to the distributed nature of the network, a different time delay is introduced on each transmitter-destination path.

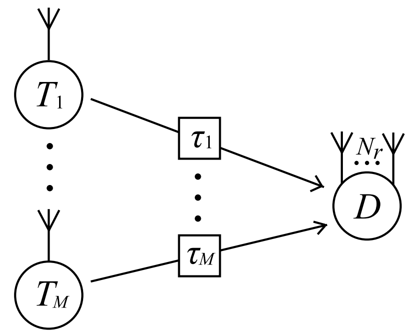

Fig. 1. Asynchronous wireless network model with M transmitters and N_r receive antennas.

$\tau_1, \tau_2, \ldots, \tau_M$ denote respectively the delays from the transmitters $T_1, T_2, \ldots, T_M$ to the destination D. We consider, for instance, the first transmitter T_1 as the node reference and we denote by $\Delta_i (i = 2, \ldots, M)$ the relative delay between the transmitter T_i and T_1: $\Delta_i = \tau_i - \tau_1$.

The fractional delays are assumed to be absorbed in multipath (cf. [18]), so the delays τ_i are integer factors of the symbol period. The delays are unknown at the transmitters, but are known at the destination. The system model is equivalent to a distributed MIMO system with M transmit antennas (one per transmitter) and N_r receive antennas.

The transmission is modeled as follows. The destination D receives the signal: $\mathbf{y} = \mathbf{H}\mathbf{X} + \mathbf{n}$, where $\mathbf{X}$ is the modulated $M \times T$ space-time codeword matrix transmitted over T symbol intervals and $\mathbf{n}$ is the Additive White Gaussian Noise (AWGN) at the destination with variance N_0. The channel is assumed to be quasi-static, so the channel matrix $\mathbf{H}$ is constant over a frame interval but is independent from one frame to another. We denote by $h_{i,j}$ the channel gain between the ith transmitter T_i and the jth antenna of D with $i = 1, \ldots, M$ and $j = 1, \ldots, N_r$.

This system model represents a general cooperative network model with the transmitters being relay nodes or base stations or mobiles able to relay etc...

In what follows, $(.)^*$, $(.)^t$ and $(.)^H$ denote respectively the conjugate, transpose and Hermitian operations.

III. Delay Tolerance of Optimal STBC

We discuss first some examples of optimal synchronous STBC to introduce the "delay tolerance" notion and after, we remind some solutions proposed to design delay tolerant codes.

A. Optimal STBC Are Not Delay Tolerant

For 2×1 MISO scheme, the Alamouti code [16] is designed and proved to achieve a diversity of two with full data rate as it transmits two symbols in two time intervals:

$$\mathbf{A_s} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

where x_1 and x_2 are the symbols to transmit; x_1^* and x_2^* are the complex conjugate of x_1 and x_2 respectively.

However, this scheme has a full diversity with two perfectly synchronized transmitters and it looses this propriety when the transmitters are not synchronized. In fact, suppose that the second transmitter T_2 has a relative delay of one symbol period ($\Delta_2 = 1$). The code matrix takes, in this case, the following form:

$$\mathbf{A_a} = \begin{bmatrix} x_1 & -x_2^* & 0 \\ 0 & x_2 & x_1^* \end{bmatrix}.$$

Thus, we have $\det(\mathbf{A_a}.\mathbf{A_a}^H) = |x_1|^2.(|x_1|^2 + 2|x_2|^2)$ which is equal to zero if only $x_1 = 0$. Thus, the imperfect delay synchronization between the two transmitters destroys the Alamouti structure and makes the destination unable to detect the original signal successfully; so the Alamouti code is not delay tolerant.

The Golden code is an optimal Space-Time code for two transmit and two receive antennas MIMO systems [17]. Its code matrix is:

$$\mathbf{G} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(x_1 + \theta x_2) & \alpha(x_3 + \theta x_4) \\ \imath\,\bar{\alpha}\,(x_3 + \bar{\theta} x_4) & \bar{\alpha}\,(x_1 + \bar{\theta} x_2) \end{bmatrix},$$

where $\imath = \sqrt{-1}$, $\theta = \frac{1+\sqrt{5}}{2}$, $\alpha = 1 + \imath(1-\theta)$, $\bar{\theta} = 1 - \theta$, and $\bar{\alpha} = 1 + \imath\,\theta$. However, it is not delay tolerant as can be seen by shifting the second row one column and then setting the entries x_1 and x_2 to zero. Similarly, the more general class of STCs, derived from cyclic division algebras (CDA) of which the Golden code is a special case, is not delay tolerant either [13].

The TAST codes were proposed in [15]. These codes provide excellent performance and flexibility with respect to signaling constellation, transmission rate, number of transmit and receive antennas, and decoder complexity. An example of TAST codes for $M = 2$ transmit and $N_r \geq 2$ receive antennas, is the TAST-2 code defined as follows [15]:

$$\mathbf{T}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 + \theta x_2 & \phi^{1/2}(x_3 + \theta x_4) \\ \phi^{1/2}(x_3 - \theta x_4) & x_1 + \theta x_2 \end{bmatrix},$$

where $\theta = e^{\imath\alpha}$ and $\phi = e^{\imath\alpha'}$.

Unfortunately, the TAST codes and other related codes available in the literature are not suitable for asynchronous cooperative communications, since they are not delay tolerant which was clearly illustrated in [13].

B. Existing Solutions for Delay Tolerant STBC

The CDA and TAST codes are not delay tolerant because they are based on threads of minimal delay ($T = M$) and hence contain diagonal matrices that are not delay tolerant. In [13], the authors extended the class of the TAST codes to the case of delay tolerant codes for cooperative diversity. Their proposed codes, named Distributed-TAST codes, are based on delay tolerant threaded structures of length growing exponentially with the number of relays. The different threads are separated by different algebraic or transcendental numbers which guarantee a nonzero determinant for the difference of every two distinct codewords. The idea is to repeat the symbols in a way that, even when the transmitters are asynchronous, the versions of the same symbols sent by the M transmitters arrive at the destination in at least M different symbol periods and thus conserve a full transmit diversity order M. Although these codes provide full-diversity gain for any delay profile, they are not minimum delay length because of symbols repetition and are no longer delay tolerant if a column of the codeword matrix is lost.

This solution was applied to the Alamouti code and a variant of the Golden code by repeating the second column [13]. For instance, the delay tolerant version of the Alamouti code is:

$$\mathbf{A_d} = \begin{bmatrix} x_1 & -x_2^* & -x_2^* \\ x_2 & x_1^* & x_1^* \end{bmatrix}.$$

In the sequel, this new form of Alamouti code will be called the Asynchronous Alamouti (AA).

Moreover, the delay tolerant version of the Golden code, that will be called hereafter the Asynchronous Golden (AG), is:

$$\mathbf{C_d} = \frac{1}{\sqrt{2(1+r^2)}} \begin{bmatrix} x_1 + \imath r x_4 & r x_2 + x_3 & r x_2 + x_3 \\ x_2 - r x_3 & \imath r x_1 + x_4 & \imath r x_1 + x_4 \end{bmatrix}, \quad (1)$$

where $r = \theta - 1$.

Although these new versions of the Alamouti code and the Golden code are delay tolerant since they achieve the maximum diversity for any shifted version of the code matrix, they suffer from a rate loss due to the repetition.

Another solution was given in [18] and [19]. The idea in these papers is to combine differently, all the symbols to send, for each antenna and each transmission; thus, even in the presence of a delay, it is sure that each symbol has at least one version that is not arriving at the same time with the other versions of the same symbol sent by the other transmitters. It was proven that these codes conserve a full rank code matrices in the asynchronous case and yet have a full diversity. A code example that satisfies these design rules is the 2×2 full-rate full-diversity space-time code mentioned in [19]:

$$\mathbf{D} = \begin{bmatrix} a x_1 + b x_2 - c x_3 - d x_4 & -c x_1 - d x_2 - a x_3 - b x_4 \\ b x_1 + a x_2 + d x_3 - c x_4 & -d x_1 + c x_2 - b x_3 + a x_4 \end{bmatrix}, \quad (2)$$

where $a = \frac{1}{\sqrt{(5+\sqrt{5})(2+\sqrt{2})}}$; $b = \frac{1}{\sqrt{(5-\sqrt{5})(2+\sqrt{2})}}$; $c = \frac{1}{\sqrt{(5+\sqrt{5})(2-\sqrt{2})}}$; $d = \frac{1}{\sqrt{(5-\sqrt{5})(2-\sqrt{2})}}$.

Suppose that the second row of the matrix $\mathbf{D}$ is shifted one column to the right due to a delay by the second transmitter of one symbol period. In this case, we still have two versions of the four symbols arriving to the destination in different time periods. This code will be referred to as the Damen Code (DC). Another 2×2 delay tolerant code was proposed in [19] and will be called here the Sarkiss Code (SC). The SC consists of building a rotated lattice in higher dimension based on the rotation matrix of the Golden code. Convenient unitary matrices to obtain the modified code were also given in [19]. However, when combining all the symbols to send, it becomes more difficult to find the optimal parameters when the number of transmitters becomes bigger. For instance, for 3×3 codes in [18], each symbol sent by a transmitter is the combination of nine information symbols.

IV. Construction Method of BDT-STBC

Here, we present a novel design to construct bounded delay tolerant (BDT) STBC from known non-delay tolerant STBC. This method is based on the concatenation of K code matrices and the permutation of the new matrix columns in a suitable manner to have a new order of the columns. The new designed code ensures a full-diversity for a set of delay profiles that can occur in the network without a rate reduction because no repetition is needed. By applying this method on optimal synchronous codes (i.e. Alamouti code, Golden code, TAST codes), we ensure the optimality of the new codes in the synchronous case and also their high performance in the asynchronous case.

A. Method Description

Let us consider a $M \times T$ code having the following matrix of index k:

$$\mathbf{X}^k = \begin{bmatrix} X_{11}^k & \dots & X_{1T}^k \\ \vdots & \ddots & \vdots \\ X_{M1}^k & \dots & X_{MT}^k \end{bmatrix}.$$

By concatenating $K \geq 2$ different matrices $\mathbf{X}^k$ ($k = 1, \ldots, K$), the new code matrix becomes:

$$\mathbf{X_c} = \begin{bmatrix} X_{11}^1 & \dots & X_{1T}^1 & \dots & X_{11}^K & \dots & X_{1T}^K \\ \vdots & \ddots & \vdots & \dots & \vdots & \ddots & \vdots \\ \underbrace{X_{M1}^1 \; \dots \; X_{MT}^1}_{\mathbf{X}^1} & & & \dots & \underbrace{X_{M1}^K \; \dots \; X_{MT}^K}_{\mathbf{X}^K} & & \end{bmatrix}.$$

Then, we permutate the columns of matrix $\mathbf{X_c}$. Let $t = 1, \ldots, T$ be the positions of the columns in matrices $\mathbf{X}^k$ ($k = 1, \ldots, K$), the new position t' of the column t of $\mathbf{X}^k$ is:

$$t' = (t-1).K + k .$$

The permutated matrix will have the following form:

$$\mathbf{X_{pc}} = \begin{bmatrix} X_{11}^1 & \dots & X_{11}^K & \dots & X_{1T}^1 & \dots & X_{1T}^K \\ \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ X_{M1}^1 & \dots & X_{M1}^K & \dots & X_{MT}^1 & \dots & X_{MT}^K \end{bmatrix}.$$

B. Method Discussion and Advantages

By using this construction method, a full transmit diversity of M is ensured for a set of delay profiles depending on the number of concatenated code matrices K. The purpose of putting several coded symbols together and permutating the code columns is to eventually prevent the versions of the same symbols from arriving to the destination at the same time in the case when a delay occurs in the network. By increasing K, we extend the number of tolerated delay profiles by the new bounded delay tolerant codes. Therefore, we can choose K with respect to the maximal delay existing in the network. However, the channel has to keep constant for longer time than other codes but this time is of the order of some symbol periods which is a feasible condition. This is a reasonable assumption in a practical network communication.

Furthermore, bounded delay tolerant codes, based on optimal known codes, have full rate in the synchronous case. When no delays exist in the communication, simple permutations, converse to the ones applied at the transmission, can be done at the receiver to decode. Hence, each initial code matrix can be decoded separately by using the known optimal decoding algorithms for these STBC (i.e., Sphere Decoder [23]). However, in the asynchronous case, the length of the received frame increase due to the delay added by the channel. Therefore, the rate decreases and the BDT codes cannot be full rate. But, the overall rate of BDT codes improves by increasing K and the BDT codes give higher rate than other codes for asynchronous communications.

V. BDT-STBC for Two Transmitters

In the case of two transmitters ($M = 2$), the optimal STBCs are the Alamouti code for $N_r = 1$ receive antenna, the TAST-2 and the Golden codes for $N_r \geq 2$ receive antennas. The construction method of BDT codes presented above will be applied to these three codes.

A. BDT-STBC Based on Alamouti Code

Let us consider the concatenation of K Alamouti code matrices $\mathbf{A}^k$ ($k = 1, \ldots, K$):

$$\mathbf{A}^k = \begin{bmatrix} x_1^k & -x_2^{*\,k} \\ x_2^k & x_1^{*\,k} \end{bmatrix}.$$

After permutation of the columns, the BDT Alamouti code matrix is:

$$\mathbf{A_{pc}} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^K & -x_2^{*\,1} & -x_2^{*\,2} & \dots & -x_2^{*\,K} \\ x_2^1 & x_2^2 & \dots & x_2^K & x_1^{*\,1} & x_1^{*\,2} & \dots & x_1^{*\,K} \end{bmatrix}.$$

A full transmit diversity of two is ensured for an interval of relative delays $\Delta = \tau_2 - \tau_1$:

$$\Delta \in \{-\Delta_{\max}, +\Delta_{\max}\},$$

with $\Delta_{\max} = K - 1$. The bounds of this interval of tolerance are deduced from the fact that the same coded symbols (i.e. $-x_2^{*\,1}$ and x_2^1) will not arrive at the same time at the destination unless the first row is shifted K columns to the left or the second row is shifted K columns to the right.

B. BDT-STBC Based on TAST-2 and Golden Codes

For clarity reasons, the TAST-2 and Golden code matrices will be written in the following manner:

$$\mathbf{X_2}^k = \begin{bmatrix} X_1^k & X_2^k \\ \phi X_2^{'\,k} & X_1^{'\,k} \end{bmatrix}, \tag{3}$$

where $X_1^k = \frac{1}{\sqrt{2}}(x_1^k + \theta x_2^k)$; $X_2^k = \frac{1}{\sqrt{2}}(x_3^k + \theta x_4^k)$; $X_2^{'\,k} = \frac{1}{\sqrt{2}}(x_3^k - \theta x_4^k)$; $X_1^{'\,k} = \frac{1}{\sqrt{2}}(x_1^k - \theta x_2^k)$,
for the TAST-2 code with $\theta = e^{\imath\alpha}$ and $\phi = e^{\imath\alpha'}$.
And $X_1^k = \frac{1}{\sqrt{5}}\,\alpha(x_1^k + \theta x_2^k)$; $X_2^k = \frac{1}{\sqrt{5}}\,\alpha(x_3^k + \theta x_4^k)$; $X_2^{'\,k} = \frac{1}{\sqrt{5}}\,\bar{\alpha}\,(x_3^k + \bar{\theta} x_4^k)$; $X_1^{'\,k} = \frac{1}{\sqrt{5}}\,\bar{\alpha}\,(x_1^k + \bar{\theta} x_2^k)$,
for the Golden code with α and θ are the Golden code parameters and $\phi = \imath$.

We apply the new construction method on the TAST-2 and Golden codes. By concatenating K code matrices, we obtain:

$$\mathbf{X_{2c}} = \begin{bmatrix} X_1^1 & X_2^1 & X_1^2 & X_2^2 & \dots & X_1^K & X_2^K \\ \phi {X'_2}^1 & {X'_1}^1 & \phi {X'_2}^2 & {X'_1}^2 & \dots & \phi {X'_2}^K & {X'_1}^K \end{bmatrix}. \quad (4)$$

After permutating the columns of this matrix, the resulting matrix of the new codes is:

$$\mathbf{X_{2pc}} = \begin{bmatrix} X_1^1 & X_1^2 & \dots & X_1^K & X_2^1 & X_2^2 & \dots & X_2^K \\ \phi {X'_2}^1 & \phi {X'_2}^2 & \dots & \phi {X'_2}^K & {X'_1}^1 & {X'_1}^2 & \dots & {X'_1}^K \end{bmatrix}. \quad (5)$$

The bounds of the tolerance interval of the BDT TAST-2 and BDT Golden codes ares the same as for the BDT Alamouti code in the previous section.

C. Rank and Determinant Criteria

In this section, we prove that the three new bounded delay tolerant, presented above and based on optimal synchronous STBCs, still respect the design criteria of space time codes even in the presence of a relative delay in a bounded interval. Actually, the BDT Alamouti code, the BDT Golden code and the BDT TAST-2 code satisfy the two following propositions.

Proposition 1: Let $\mathbf{X}$ *be the transmitted codeword, and* $\mathbf{T}$ *be the erroneously decoded codeword at the destination. The determinant* $\det(\mathbf{A}) = \det((\mathbf{X}-\mathbf{T}).(\mathbf{X}-\mathbf{T})^H)$ *is non-zero for all the values of relative delays* Δ *in the interval of tolerance. Thus, the BDT code matrix has full rank for these values of* Δ*.*

The proof of Proposition 1 is drawn in Appendix A.

Proposition 2: The pairwise error probability $\mathbb{P}(\mathbf{X} \to \mathbf{T})$ *of the new BDT code for* Δ *in the interval of tolerance can be upper bounded by:*

$$\mathbb{P}(\mathbf{X} \to \mathbf{T}) \leq \left(\prod_{i=1}^{2} \lambda_i\right)^{-N_r} \left(\frac{1}{8N_0}\right)^{-2N_r}, \quad (6)$$

where λ_1 *and* λ_2 *are the eigenvalues of matrix* $\mathbf{A}$*.*

The proof of Proposition 2 is omitted here for lack of space, but a similar proof can be found in [20].

In Equation (6), the term $\frac{1}{8N_0}$ represents the Signal to Noise Ratio (SNR) of the network. The diversity order d can be deduced from the PEP as the exponent of the SNR [1]; thus, it is equal to $2.N_r$ for any relative delay Δ in the interval of tolerance.

D. Minimal Determinant of BDT Golden and BDT TAST-2 Codes

Let us compare the performances of the BDT Golden and BDT TAST-2 codes for different delay configurations that can exist in a cooperative network. For the BDT TAST-2 code, we consider the optimal parameters indicated in [15] and which are: $\theta = e^{\imath\frac{\pi}{4}}$ and $\phi = e^{\imath\frac{\pi}{6}}$.

Table I contains the value of the minimal determinant values for BDT Golden code and BDT TAST-2 code for 4-QAM and 16-QAM symbols.

Comparing the minimal determinants of the two codes, we conclude the following:

TABLE I
MINIMAL DETERMINANT VALUES FOR BDT GC AND BDT TAST-2

		4-QAM	16-QAM
BDT GC	Synchronous	0.8	0.032
	Asynchronous	0.8	0.032
BDT TAST-2	Synchronous	0.268	0.011
	Asynchronous	1	0.04

- For the synchronous case, the BDT Golden code achieves higher minimal determinant than the BDT TAST-2 code as it is the case when comparing the Golden code and the TAST-2 code.
- On the contrary, when the network is asynchronous, the minimal determinant of the BDT TAST-2 code is bigger than the BDT Golden code.

E. Numerical Results

The simulations results show error rates as a function of E_b/N_0 in dB which is adjusted as follows:

$$\left.\frac{E_b}{N_0}\right|_{dB} = \left.\frac{E_s}{N_0}\right|_{dB} - 10\log_{10} R ,$$

where E_s is the average signal energy per receive antenna and R is the code rate in bit per channel use (bpcu). Frame Error Rate (FER), Symbol Error Rate (SER) and Bit Error Rate (BER) are used as performance comparison metrics between the different codes and the exhaustive Maximum Likelihood (ML) is used for detection at the destination. For the simulations, R is considered to be the rate at the transmitters which is calculated in the synchronous case and does not take into account the added delay by the channels after transmission.

First let us compare the BDT Alamouti code (BDT AC) with the Asynchronous Alamouti (AA) for single antenna destination ($N_r = 1$). The BDT AC with $K = 3$ concatenated Alamouti code matrices is considered. To have the same spectral efficiency of $R = 2$ bpcu, BDT AC sends 4-QAM symbols x_i and AA sends 8-PSK symbols. In Figures 2 and 3, FER, SER and BER curves are given for BDT AC and AA for $\Delta = 0$ and $\Delta = 1$ respectively. It can noticed that the BDT AC outperforms the AA for both synchronous and asynchronous cases; this is due mainly to the symbols repetition in the AA.

Next, we compare the BDT Golden code (BDT GC) and the BDT TAST-2 code (BDT TAST-2) with other delay tolerant codes for $M = 2$ transmitters and $N_r = 2$ antennas at the destination. For the BDT codes, $K = 2$ matrices are concatenated and 4-QAM symbols S_i are used for a data rate $R = 4$ bpcu. Figures 4, 5 and 6 show the FER and BER of the BDT codes designed for 2×2 cooperative systems and of the existing delay-tolerant codes: Asynchronous Golden (AG), Damen Code (DC) and Sarkiss Code (SC). Our proposed codes outperform almost all the time the latter codes for $\Delta = 0$ and $\Delta = 1$. Actually, the SC gives better FER than the BDT GC because the frame length of our code is bigger than the SC which causes more erroneous frames. It is also worth to

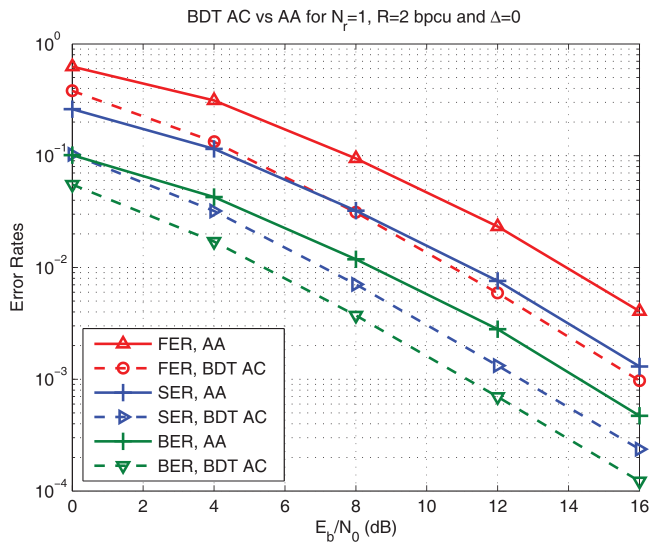

Fig. 2. Error rates comparison between BDT AC and AA for $\Delta = 0$.

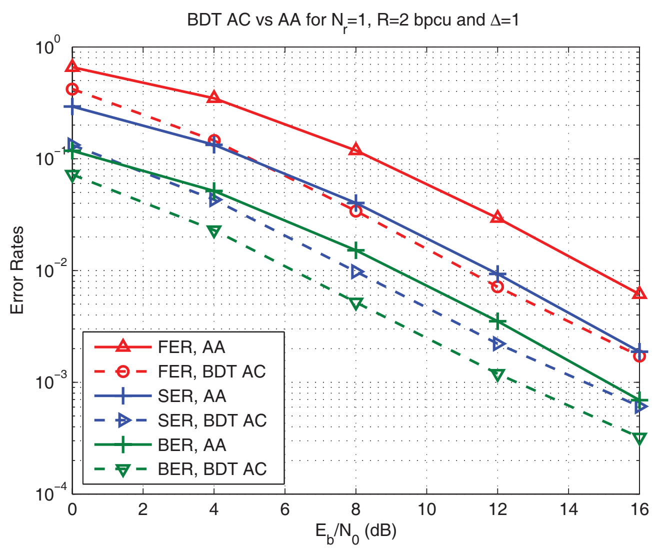

Fig. 3. Error rates comparison between BDT AC and AA for $\Delta = 1$.

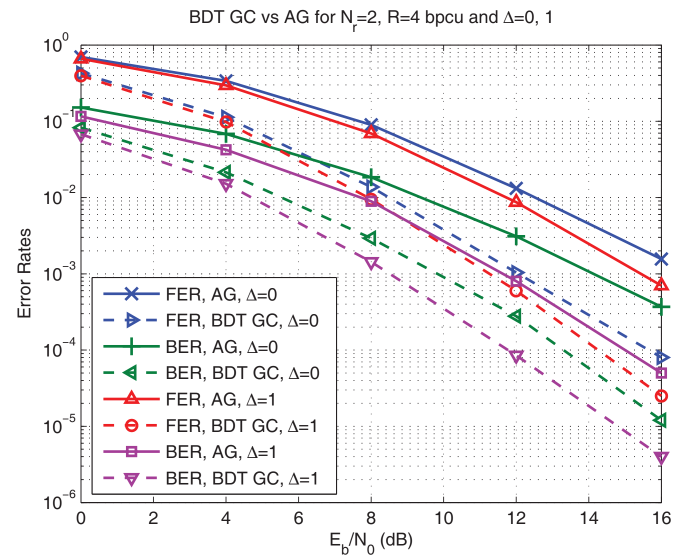

Fig. 4. Error rates comparison between BDT GC and AG.

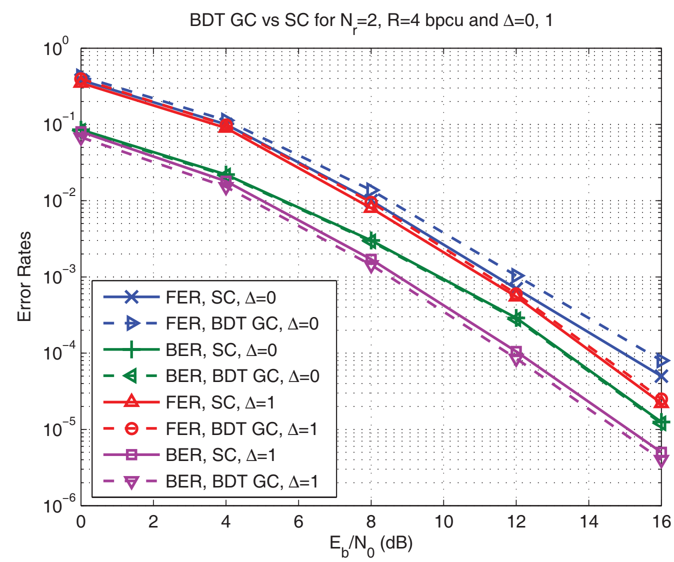

Fig. 5. Error rates comparison between BDT GC and SC.

note that even if the data rate R is equal for all codes in the synchronous case, however for $\Delta = 1$ the rate of our BDT codes become higher due to their bigger frame length compared to other codes.

VI. BDT-STBC FOR $M > 2$ TRANSMITTERS

When more than two transmitters are in the network, an optimal Threaded Algebraic Space Time (TAST) code can be used as the base code in order to construct BDT codes.

We first recall the general definition of a TAST code. The transmitted symbols of a TAST code are finitely generated from an underlying finite constellation using algebraic number field constructions [13] [15]. Let us denote $\mathcal{A}$ the multidimensional constellation considered (QAM, PAM, etc), and $\mathbb{F} = \mathbb{Q}(\imath)$ the field of complex rational numbers. $\mathbb{F}(\theta)$ is an extension field of degree $n = [\mathbb{F}(\theta) : \mathbb{F}]$ with θ an algebraic number of degree n.

For an arbitrary number of threads L, the TAST codes are constructed by transmitting a scaled Diagonal Algebraic Space Time (DAST) code in each thread. For instance, $\mathbf{u}_j = \phi_j\, \mathbf{s}_j = \phi_j\, \mathbf{M}_j\, \mathbf{x}_j$ is transmitted over thread $l_j (j = 1, \ldots, L)$, where $\mathbf{x}_j = (x_{j1}, \ldots, x_{jM})^t$ is the information symbol vector sent over thread l_j and $x_{jk} \in \mathcal{A}$.

Matrix $\mathbf{M}_j$ is an $M \times M$ real or complex rotation that achieves full diversity as a DAST code and is constructed on the algebraic number field $\mathbb{F}(\theta)$ [24] [25]. The complex numbers $\phi_j (j = 1, \ldots, L)$ are chosen to ensure full diversity and maximize the coding gain for the composite code by separating the threads over different algebraic subspaces.

A TAST code is said to be symmetric if the same DAST code is used in all the threads and thus all the rotation matrices are the same $(\mathbf{M}_1 = \ldots = \mathbf{M}_L = \mathbf{M})$.

Hereafter, we prove that a full diversity order is always ensured when the new design method is applied to the TAST Codes for a bounded number of delay profiles. And an example of the new design method based on a TAST code for three transmitters is also given.

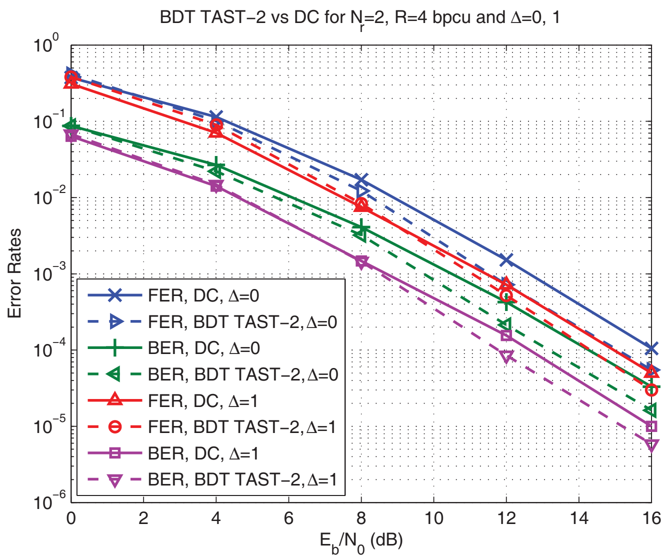

Fig. 6. Error rates comparison between BDT TAST-2 and DC.

A. Rank Criterion

The BDT TAST codes satisfy the following proposition:

Proposition 3: When considering a TAST code as the base code with the generalized condtruction method of BDT codes, the determinant of the difference matrix of any two distinct codewords is always not null for a set of relative delays $(\Delta_2, \ldots, \Delta_M)$. *Thus, the code matrix is full rank and the BDT TAST code ensures full diversity for these delays profiles.*

The proof of Proposition 3 is given in Appendix B.

B. BDT TAST Code for $M = 3$ transmitters

The symmetric TAST code for $M = L = 3$ and $N_r \geq 3$ receive antennas, called here the TAST-3 code, has the following matrix:

$$\mathbf{S}^k = \begin{bmatrix} s_{11}^k & \phi^{\frac{2}{3}} s_{32}^k & \phi^{\frac{1}{3}} s_{23}^k \\ \phi^{\frac{1}{3}} s_{21}^k & s_{12}^k & \phi^{\frac{2}{3}} s_{33}^k \\ \phi^{\frac{2}{3}} s_{31}^k & \phi^{\frac{1}{3}} s_{22}^k & s_{13}^k \end{bmatrix},$$

where $(s_{j1}^k, s_{j2}^k, s_{j3}^k)^t = \mathbf{M}.(x_{j1}, x_{j2}, x_{j3})^t$ with j the thread number $(j = 1, 2, 3)$. $\mathbf{M}$ is an optimal 3×3 algebraic rotation matrix and $x_{11}, \ldots, x_{33}$ are the information symbols belonging to $\mathcal{A}$. ϕ is an appropriate algebraic or transcendental number chosen such that the numbers $\{1, \phi^{\frac{1}{3}}, \phi^{\frac{2}{3}}\}$ are algebraically independent over the algebraic number field $\mathbb{F}(\theta)$ that contains the elements of the rotation matrix $\mathbf{M}$ [15].

Next, we give an example of the BDT construction method on $K = 3$ TAST-3 codes $\mathbf{S}^1$, $\mathbf{S}^2$ and $\mathbf{S}^3$. Thus, the BDT TAST-3 code has the following matrix for $K = 3$:

$$\mathbf{S_{pc}} = \begin{bmatrix} s_{11}^1 & s_{11}^2 & s_{11}^3 & \ldots & \phi^{\frac{1}{3}} s_{23}^1 & \phi^{\frac{1}{3}} s_{23}^2 & \phi^{\frac{1}{3}} s_{23}^3 \\ \phi^{\frac{1}{3}} s_{21}^1 & \phi^{\frac{1}{3}} s_{21}^2 & \phi^{\frac{1}{3}} s_{21}^3 & \ldots & \phi^{\frac{2}{3}} s_{33}^1 & \phi^{\frac{2}{3}} s_{33}^2 & \phi^{\frac{2}{3}} s_{33}^3 \\ \phi^{\frac{2}{3}} s_{31}^1 & \phi^{\frac{2}{3}} s_{31}^2 & \phi^{\frac{2}{3}} s_{31}^3 & \ldots & s_{13}^1 & s_{13}^2 & s_{13}^3 \end{bmatrix}.$$

The full diversity of $3.N_r$ is guaranteed for the synchronous case and for a set of relative delay profiles ($\Delta_2 = \tau_2 - \tau_1, \Delta_3 = \tau_3 - \tau_1$). Basically, the tolerance interval for Δ_2 and Δ_3 is {-1, 0, +1}; but other delay profiles are also possible.

TABLE II
TOLERABLE RELATIVE DELAY PROFILES FOR $M = T = 3$

Δ_i	Δ_j
0	{-2, -1, 0, 1, 2}
1	{-1, 0, 1, 2}
-1	{-2, -1, 0 ,1}
2	{0, 1, 2}
-2	{-2, -1, 0}
$(\Delta_2, \Delta_3) = (-1, -3); (-2, -4); (3, 1); (4, 2)$	

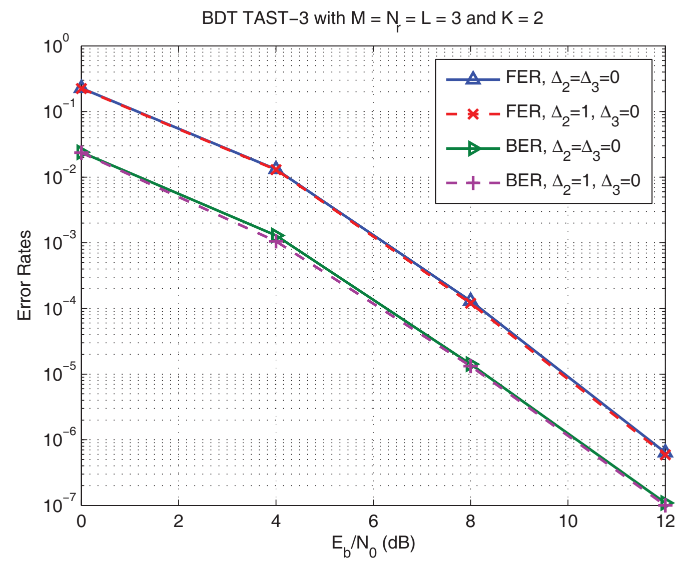

Fig. 7. Error rates of BDT TAST-3 for $M = N_r = 3$ and $K = 2$.

Table II provides the different relative delay profiles for which the new code ensures the full diversity: Δ_i corresponds to one of the two relative delays (Δ_2 or Δ_3) and Δ_j to the other one.

C. Numerical Results

Let us consider the TAST code presented is Section VI-B for three transmitters ($M = L = 3$) and with three receiving antennas ($N_r = 3$). Two code matrices ($K = 2$) are concatenated and permutated in order to form the BDT TAST-3 code.

The same decoding configurations are used as in Section V-E. In Figure 7, the FER and BER are plotted for synchronous case ($\Delta_2 = \Delta_3 = 0$) and an asynchronous case ($\Delta_2 = 1, \Delta_3 = 0$).

VII. Conclusions

Bounded delay tolerant STBC were introduced in this paper. These codes ensure a full diversity for a synchronous network and for a set of relative delays between the transmitters and have optimal rates and better error performance than known delay tolerant codes. The design concept of such codes are simple and based on optimal synchronous STBCs. Theoretical proofs that the new family of codes respect the STBC design criteria were also given.

Appendix A

The following assumptions are considered to make the derivations clearer without loss of generality:

- We consider that $\tau_2 \geq \tau_1$ and hence the relative delay Δ is positive: $\Delta = \tau_2 - \tau_1 \geq 0$.
- The destination is considered to be synchronous with relay T_1 so that $\tau_1 = 0$.

Next, we give the proof for the BDT TAST-2 and BDT Golden codes. In fact, for the BDT Alamouti code, the proof is very similar.
At the destination, the received signal $\mathbf{y}$ can be written as:

$$\mathbf{y} = \mathbf{H}\,\mathbf{X_{2pc}} + \mathbf{n},$$

where

$$\mathbf{X_{2pc}} = \begin{bmatrix} X_1^1 & \dots & X_1^K & X_2^1 & \dots & X_2^K & \mathbf{0}^\Delta \\ \mathbf{0}^\Delta & \phi X_2^{'1} & \dots & \phi X_2^{'K} & X_1^{'1} & \dots & X_1^{'K} \end{bmatrix};$$

$$\mathbf{y} = \begin{bmatrix} y_{1,1} & y_{2,1} & \cdots & y_{(2K+\Delta),1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1,N_r} & y_{2,N_r} & \cdots & y_{(2K+\Delta),N_r} \end{bmatrix};$$

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{2,1} \\ \vdots & \vdots \\ h_{1,N_r} & h_{2,N_r} \end{bmatrix};$$

$$\mathbf{n} = \begin{bmatrix} n_{1,1} & n_{2,1} & \cdots & n_{(2K+\Delta),1} \\ \vdots & \vdots & \ddots & \vdots \\ n_{1,N_r} & n_{2,N_r} & \cdots & n_{(2K+\Delta),N_r} \end{bmatrix}.$$

Let $\mathbf{T}$ be the erroneously decoded codeword at the receiver; thus

$\mathbf{X_{2pc}} - \mathbf{T} = \begin{bmatrix} e_1^1 & \dots & e_1^K & e_2^1 & \dots & e_2^K & \mathbf{0}^\Delta \\ \mathbf{0}^\Delta & \phi e_2^{'1} & \dots & \phi e_2^{'K} & e_1^{'1} & \dots & e_1^{'K} \end{bmatrix}$ is the codeword difference matrix, with $e_i^k = X_i^k - T_i^k$, $e_i^{'k} = X_i^{'k} - T_i^{'k}$, and $i = 1, 2$.

Matrix $\mathbf{A}$ will be defined as: $\mathbf{A} = (\mathbf{X_{2pc}} - \mathbf{T}).(\mathbf{X_{2pc}} - \mathbf{T})^H$. Hereafter, we prove that $\det(\mathbf{A})$ is different from zero for different values of Δ_2 in the interval of tolerance.

1) Synchronous Case ($\Delta_2 = 0$): By making converse permutations on matrix $\mathbf{X_{2pc}}$ (5), we reobtain $\mathbf{X_{2c}}$ (4) the concatenation of the K matrices $\mathbf{X_2}^k$ (3). It is known that $\det(\mathbf{X_{2pc}}^k) \neq 0$ for TAST-2 code and Golden code. Since $\left(\mathbf{X_2}^k \ \mathbf{X_2}^{k\,H}\right)$ are positive definite matrices, we use the determinant inequality in [26]:

$$\begin{aligned} \det\left(\mathbf{X_2}_c\,\mathbf{X_2}_c^H\right) &= \det\left(\sum_{k=1}^{K}\left(\mathbf{X_2}^k\,\mathbf{X_2}^{k\,H}\right)\right) \\ &\geq \min_{\mathbf{X_2}_c \neq \mathbf{0}^{2\times 2K}} \sum_{k=1}^{K} \det\left(\mathbf{X_2}^k\,\mathbf{X_2}^{k\,H}\right). \end{aligned} \tag{7}$$

By considering the codeword difference matrices, it is easy to deduce from (7) the following:

$$\det(\mathbf{A}) \geq \min \sum_{k=1}^{K} \det\left(\mathbf{B}^k\,\mathbf{B}^{k\,H}\right);$$

$$\text{with } \mathbf{B}^k = \begin{bmatrix} e_1^k & e_2^k \\ \phi e_2^{'k} & e_1^{'k} \end{bmatrix}, \text{ and } \det\left(\mathbf{B}^k\,\mathbf{B}^{k\,H}\right) \geq 0.$$

Because $\mathbf{X_{2pc}}$ and $\mathbf{T}$ are two different codewords, $\det\left(\mathbf{B}^\ell \mathbf{B}^{\ell^H}\right) \neq 0$ is verified for at least one value of k, and $\det(\mathbf{A}) = \det\left((\mathbf{X_{2pc}} - \mathbf{T}).(\mathbf{X_{2pc}} - \mathbf{T})^H\right)$ cannot be equal to zero.

2) Asynchronous Case ($\Delta_2 \neq 0$): In the presence of a delay, the code matrix cannot be written as a concatenation of 2×2 code matrices as for the synchronous case.
Thus, we derive the determinant of matrix $\mathbf{A}$ for the non-zero relative delays in the interval of tolerance ($0 < \Delta_2 < K$) with the assumptions considered here.

We have: $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where:

- $a_{11} = \sum_{i=1}^{2}\left(\sum_{k=1}^{K} |e_i^k|^2\right)$ and $a_{22} = \sum_{i=1}^{2}\left(\sum_{k=1}^{K} |e_i^{'k}|^2\right)$;
- $a_{12} = \sum_{k=1}^{K-\Delta}(e_1^{k+\Delta})(\phi e_2^{'k})^* + \sum_{k=1}^{\Delta}(e_2^k)(\phi e_2^{'K-\Delta+k})^* + \sum_{\Delta+1}^{K}(e_2^k)(e_1^{'k-\Delta})^*$ and $a_{21} = a_{12}^*$.

By manipulating the terms of $\det(\mathbf{A})$ conveniently, the determinant can be written as the sum of two groups of terms:

- The first group terms are products of squared modulus: $|e_i^k|^2\,|e_i^{'k}|^2$ with $i = 1, 2$ and $k = 1, \dots, K$.
- The terms in the second group have the following form: $|m|^2 + |n|^2 - 2\,\mathcal{R}(p.\,m^*.\,n)$.
  $\mathcal{R}(x)$ being the real part of the complex number x.
  m and n are products of e_i^k and $e_i^{'k}$; p can have one of these values $\{1, \phi, \phi^*\}$.

The terms in the two groups above are positive and so $\det(\mathbf{A})$ is a sum of positive terms that cannot be all null at the same time for two different codewords $\mathbf{X_{2pc}}$ and $\mathbf{T}$.

Appendix B

Consider the complex numbers separating the different threads $\phi_j = \phi^{j-1}$ ($j = 1, \dots, L$) so that $\{1, \phi, \dots, \phi^{L-1}\}$ are algebraically independent over $\mathbb{F}(\theta)$. We concatenate K TAST code matrices $(\mathbf{S}^1, \dots, \mathbf{S}^K)$ of size $M \times M$ each. After permutation of the columns, $\mathbf{S}$ denotes the BDT TAST code $M \times MK$ matrix. The new BDT code is also constituted by L threads and can be written as:

$$\mathbf{S} = p(\mathbf{s}_1) + \phi\,p(\mathbf{s}_2) + \dots + \phi^{L-1}\,p(\mathbf{s}_L), \tag{8}$$

where $p(\mathbf{s}_j)$ represents the thread formed by the concatenation and permutation of the thread $\mathbf{s}_j$ ($j = 1, \dots, L$) of the K TAST codes.

For instance, $p(\mathbf{s}_1)$ of size $M \times MK$ has the following form:

$$\begin{bmatrix} s_{11}^1 & \dots & s_{11}^K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & s_{12}^1 & \dots & s_{12}^K & 0 & 0 & 0 & 0 & \dots & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & s_{1M}^1 & \dots & s_{1M}^K \end{bmatrix}.$$

The position of symbols s_{ji}^k ($k = 1, \dots, K$ and $i = 1, \dots, M$) of other $p(\mathbf{s}_j)$ can be deduced by K cyclic left shifting to the lines of $p(\mathbf{s}_{j-1})$.

Let us consider the difference $\mathbf{dS}$ between two distinct codewords $\mathbf{S}$ and $\mathbf{S}'$:

$$\mathbf{dS} = \mathbf{S} - \mathbf{S}' = \sum_{m=1}^{L} \phi^{m-1}\Big(p(\mathbf{s}_m) - p(\mathbf{s'}_m)\Big).$$

Because $\mathbf{S}$ and $\mathbf{S}'$ are distinct codewords, they have at least one different TAST code matrix $\mathbf{S}^q$ with ($q \in \{1, \dots, K\}$). Let ℓ denote the largest index for which $s_{\ell i}^q \neq s'^q_{\ell i}$ but $s_{\ell i}^q = s'^q_{\ell i}$ for $m > \ell$. Then:

$$\mathbf{dS} = \sum_{m=1}^{\ell} \phi^{m-1}\Big(p(\mathbf{s}_m) - p(\mathbf{s'}_m)\Big).$$

To guarantee that the new code matrix is full rank, it suffices to verify that there exists a square matrix $M \times M$ that is full rank, i.e., its determinant is non-zero.
We identify columns $c_1, \dots, c_M$ in $\mathbf{dS}$ that together form a submatrix $\mathbf{SM}$ whose maximal-ϕ entries of $\mathbf{S}^q$ (those that are multiples of $\phi^{\ell-1}$) occupy the principal diagonal, i. e., the diagonal values are: $\phi^{\ell-1}\big(s_{\ell i}^q - s'^q_{\ell i}\big)$ for $i = 1, \dots, M$.

For the synchronous case, this submatrix $\mathbf{SM}$ is the difference TAST code matrix $\mathbf{dS}^q$ between the two different $M \times M$ TAST codes $\mathbf{S}^q$ and $\mathbf{S}'^q$ which has a non-zero determinant.
For the asynchronous cases, the submatrix $\mathbf{SM}$ determinant is:

$$D(\phi) = G(\phi) + \phi^{M(\ell-1)} \prod_{i=1}^{M} \big(s_{\ell i}^q - s'^q_{\ell i}\big),$$

where $G(\phi)$ is a polynomial in ϕ over $\mathbb{F}(\theta)$ of degree n_G.
By code design and for the values of $(\Delta_2, \dots, \Delta_M)$ in the set of tolerable delay profiles, the degree of $G(\phi)$ is $n_G < M(\ell-1)$ and thus $D(\phi)$ is a non trivial polynomial in ϕ of degree $n_D = M(\ell-1)$ over $\mathbb{F}(\theta)$. Because ϕ is not the root of any nontrivial polynomial of degree $< M(M-1)$ over $\mathbb{F}(\theta)$; hence $D(\phi) \neq 0$ and the matrix $\mathbf{dS}$ is of full rank which concludes our proof.

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