Asynchronous Full-Diversity High-Rate Coding Scheme for Cooperative Relay Networks

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***Abstract*—In previous works on two-relay two-hop asynchronous cooperative networks we showed that, based on outage probability derivation, sending the same information frame by the two relays with a relative delay ensures a diversity order of two. In this paper, we propose a new practical asynchronous distributed coding scheme. This code performs a combination of the symbols with constellation rotation and gives better error rate performance than other codes. We also prove that it achieves full-diversity and high-rate.**

I. Introduction

Cooperative communication can be used as an alternative technique of the multiple-antenna (MIMO) systems to provide spatial diversity for networks in which nodes cannot have more than one antenna due to size, cost, or hardware limitations. Recently, many distributed Space-Time Codes following the well-known rank and determinant criteria were designed to provide specific diversity and coding gain by assuming perfect synchronization among the cooperative nodes or relays. But due to the distributed nature of the cooperative networks, perfect synchronization is difficult, if not impossible, to achieve among the relays. The lack of perfect delay synchronization among the cooperative transmitting nodes destroys the required Space-Time Code (STC) signal structure, and prevents the transmitted symbols from being successfully detected at the receiver.

Many recent works have proposed solutions to preserve the diversity order when the relays have arbitrary relative delays in transmitting their symbols; for instance, the Orthogonal Frequency Division Multiplexing (OFDM) technique, or the design of a Space-Time Bloc Code (STBC) that is delay tolerant i.e. the code matrix remains full rank even in the presence of a delay between the relays. In [1], to reduce the impact of synchronization errors, the authors proposed to use Space-Time Codes designed for frequency-selective channels to combat these errors; in particular, Time-Reverse Space-Time Code (TR-STC) and Space-Time OFDM (ST-OFDM) were considered. The authors of [2] designed a distributive Space-Frequency code based on OFDM for frequency selective fading channels. Cyclic prefix is used at the relays to combat the timing errors and the delay of multipath. However, these solutions are not suitable for non OFDM systems and they cause a rate loss that results from adding cyclic prefix.

On the other hand, many delay tolerant codes were proposed. In [3], the author showed that codes obtained form generalization of the construction in [4] preserve the diversity gain despite the timing offset among the relay nodes. He also showed that certain binary STBC derived from the stacking construction [5] are delay tolerant. In [6], the authors build on the framework provided by [7] and [8] to design a new class of STC that shares the advantages of the Threaded Algebraic Space-Time (TAST) but are also delay tolerant. They refer to the new codes as "Distributed TAST codes". However, to successfully decode these codes, we should have a sufficient number of antennas at the receiver which means that single antenna nodes cannot use these codes.
The Alamouti code [9] is optimal for a 2×1 MIMO system but loses its optimality in the presence of asynchronism. In [10], the authors addressed the synchronization problem by developing a new receiving scheme for Alamouti's STBC cooperative transmission with two asynchronous transmitting nodes. This new receiving scheme, based on linear prediction, can tolerate the delay asynchronism. And in [6], a solution was given to make the Alamouti code delay tolerant.

In this paper, we propose a new 2×1 coding scheme for asynchronous relay networks with single antenna nodes. Indeed, in a previous work [11], an asynchronous two-relay two-hop network with one antenna per node was considered. It was shown that if the relays send the same frame of symbols in a delayed manner, a full diversity order of two is reached. But this method is not optimal from the rate point of view, so we will be able to increase the rate by using the new asynchronous code that carries out a symbol combination with constellation rotation. By calculating the pairwise error probability (PEP), it will be proven that the proposed code has a full rank for any non-zero relative delay and thus, it has a diversity of two. Besides, the code parameter is optimized.

The paper is organized as follows. In Section II, the asynchronous relay network is described and a comparison with the synchronous case is given. The delay tolerance of the Alamouti code is also discussed. In Section III, we give the structure of the new asynchronous code and proof that it verifies the rank and determinant criteria. Some examples of the new code are given in Section IV and their performances are compared with other codes. In Section V, we give conclusions.

II. System Model and Background

We consider a wireless network that consists in a source S, a destination D, and M relay nodes. All the nodes of the network have a single antenna, so the relays work in the half-duplex mode, which prohibits them from transmitting and receiving at the same time. We assume that the source-destination link is very bad and quasi nonexistent. The Decode-and-Forward (DF) cooperative protocol is used in the relay network. The transmission period is divided into two consecutive phases. In the first phase, the source broadcasts its message during the first $\frac{T}{2}$ channel uses. In the second phase, the source stops transmitting and the relays that decode successfully the received signals, send their symbols to the destination in the remaining $\frac{T}{2}$ channel uses. In what follows, for the Phase II, two relays that were able to decode the received signals without errors are selected and named R_1 and R_2. The network model is shown in Figure 1. Due to the distributed nature of the network, a different time delay is introduced on each relay-destination path. τ_1 and τ_2 denote respectively the delays from the relays R_1 and R_2 to the destination D, and the relative delay Δ between the two relays is equal to: $\Delta = \tau_2 - \tau_1$.

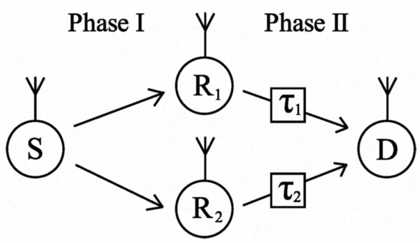

Fig. 1. The asynchronous two-relay two-hop wireless network model.

The fractional delays are assumed to be absorbed in multipath (cf. [6]), so the delays τ_1, τ_2 and Δ are integer factors of the symbol period. The delays are unknown at the relays, but are known at the destination. The system model is similar to the usual MISO system with two transmit antennas (one per relay) and one receive antenna at the destination.

Suppose that the source S transmits the following N-symbol frame $\mathbf{S} = [S_1, S_2, \ldots, S_N]$ using the codebook C. The relays R_1 and R_2 successfully decode the entire frame. The symbols S_i $(i = 1, \ldots, N)$ belong to a certain constellation (M-QAM). During the second phase, the relays send the modulated signal $\mathbf{S}$ using the codebook C, and the destination receives:

$$\mathbf{y} = \mathbf{H}\,\mathbf{S} + \mathbf{n}\,,$$

$\mathbf{n}$ is the Additive White Gaussian Noise (AWGN) at the destination D with variance N_0. The channels are assumed to be quasi-static, so the channel transfer matrix $\mathbf{H}$ is constant over a frame interval but is independent from one frame to another. We denote by h_1 and h_2 the channel gains between the relays R_1 and R_2 respectively and the destination D.

For instance, when $\tau_2 > \tau_1$, the following frames will be received at the destination D from the two relays:

$$\begin{array}{lllllllll} R_1: & \mathbf{0}^{\tau_1} & S_1 & \ldots & S_i & \ldots & S_N & \mathbf{0}^{\Delta} \\ R_2: & \mathbf{0}^{\tau_2} & \ldots & S_1 & \ldots & S_i & \ldots & S_N \end{array}$$

$\mathbf{0}^{\tau}$ denotes an all-zero vector of length τ.

This scheme of transmitting symbols by the two relays will be called the Naive Scheme (NS). In this case, we need $(2N + \tau_{max})$ symbol periods to send the N symbol frame from the source to the destination; τ_{max} being the maximum of the delays τ_1 and τ_2. Because the delays in a network are limited to a few symbol periods, by taking a long information frame, we can assume that $N \rightarrow \infty$ in the derivation of the rate r_1 of the NS. Thus, we have:

$$r_1 = \lim_{N\rightarrow\infty} \frac{N}{2N + \tau_{max}} \cong \frac{N}{2N} = \frac{1}{2}.$$

So, the rate of the Naive Scheme is not optimal and we can achieve a higher rate in the network.

A. Synchronous vs Asynchronous Schemes

It is interesting to compare the performance of the NS with the classical synchronous DF protocol that requires accurate time synchronization between the relays and different codebooks in the second phase. For the synchronous DF, the relays R_1 and R_2 use respectively the independent codebooks C^1 and C^2 in Phase II to encode the source signal $\mathbf{S}$ sent in Phase I and they transmit respectively the frames $[S_1^1, S_2^1, ..., S_N^1]$ and $[S_1^2, S_2^2, ..., S_N^2]$ synchronously to D.

Figure 2 gives the outage probabilities of the NS and the synchronous DF. It is clear that the diversity order of the NS is equal to two when the relative delay $\Delta \neq 0$ and it becomes equal to one if $\Delta = 0$. Also the NS achieves nearly the same outage probability performance as the synchronous DF when $\Delta \neq 0$.

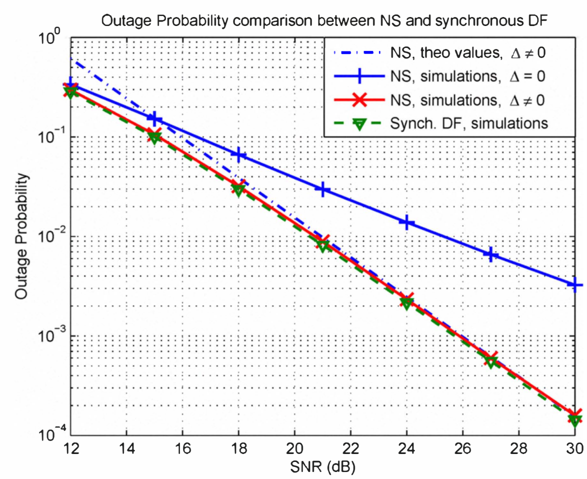

Fig. 2. Outage probabilities of the NS and the synchronous DF protocol.

B. Alamouti Code in Asynchronous Case

The Alamouti code achieves a diversity of two with full data rate as it transmits two symbols in two time intervals:

$$\mathbf{A_s} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

where x_1 and x_2 are the symbols to transmit; x_1^* and x_2^* are the complex conjugate of x_1 and x_2 respectively.

Initially, it was designed for MISO systems with two transmit antennas and one receive antenna; but a distributive version of the code can be used in relay networks. In this case, the source transmits the symbols x_1 and x_2 in the first phase, and the relays use the Alamouti scheme to transmit in the second phase. For instance, relay R_1 sends the first line of matrix $\mathbf{A_s}$ and relay R_2 sends the second line.

The Alamouti scheme has a full diversity with two perfectly synchronized transmitting relays but loses this propriety when the relays are not synchronized. In fact, suppose that the second relay R_2 has a delay of one symbol period ($\tau_2 = 1$). The code matrix takes, in this case, the following form:

$$\mathbf{A_a} = \begin{bmatrix} x_1 & -x_2^* & 0 \\ 0 & x_2 & x_1^* \end{bmatrix}.$$

By considering $\mathbf{A_a}^H$ the Hermitian transpose of $\mathbf{A_a}$, we have $\det(\mathbf{A_a}.\mathbf{A_a}^H) = |x_1|^2 \left(|x_1|^2 + 2\,|x_2|^2\right)$ which is equal to zero if only $x_1 = 0$. Thus, the imperfect delay synchronization between the two relays destroys the Alamouti structure and makes the destination unable to detect the original signal successfully; so the Alamouti code is not delay tolerant.

An extension of the Alamouti scheme was given in [6] to make the code delay tolerant:

$$\mathbf{A_d} = \begin{bmatrix} x_1 & -x_2^* & -x_2^* \\ x_2 & x_1^* & x_1^* \end{bmatrix}.$$

In the sequel, this new form of Alamouti code will be called the Asynchronous Alamouti (AA).

III. New Asynchronous Code

As shown in Section II, for an asynchronous network, sending the source information by the two relays gives a diversity of two. Although the NS outage probability performance approach that of the synchronous protocol, its rate is not optimal. Therefore, we design a new coding scheme that ensures a full diversity for asynchronous relay networks like the NS but which gives also a higher rate and better error rate performance than other codes.

The structure of the asynchronous code is given in Section III-A. We also prove that the new code verifies the rank and determinant criteria in Sections III-B and III-C by calculating the pairwise error probability (PEP).

A. Code Structure

To increase the rate, we propose the following code scheme:

$$\begin{array}{lcccccc} R_1: & X_1 & X_2 & \dots & X_i & \dots & X_{\frac{N}{2}} \\ R_2: & X_1' & X_2' & \dots & X_i' & \dots & X_{\frac{N}{2}}' \end{array}$$

$X_i = \frac{1}{\sqrt{2}}\left(S_{2i-1} + \theta\, S_{2i}\right)$ and $X_i' = \frac{1}{\sqrt{2}}\left(S_{2i-1} - \theta\, S_{2i}\right)$, where $\theta = e^{\imath\alpha}$ and $i = 1, \dots, \frac{N}{2}$; N being an even number. In fact, the use of θ will result in a constellation rotation. Each pair of points of the constellation of the symbols S_j ($j = 1, \dots, N$) will give unique combined symbols X_i and X_i' but which are two different points of a new bigger constellation. Due to this bijective mapping, it will be possible to decode the combined symbols. The proposed coding scheme creates also a diversity of constellation because each relay will code the two symbols (S_{2i-1} and S_{2i}) by a different point of the new constellation. The choice of the parameter α of θ will be discussed later.

We calculate the rate of this coding scheme that we will call the Combination Code (CC). The source sends the combined symbols $[X_1, X_2, \dots, X_{\frac{N}{2}}]$ in Phase I, and thus it needs $\frac{N}{2}$ symbol periods to send the N symbols S_i. In Phase II, the relays need $(\frac{N}{2} + \tau_{max})$ symbol periods to transmit the information to the destination. This way, $(N + \tau_{max})$ symbol periods are needed to send the N symbols. The rate of CC is equal to:

$$r_2 = \lim_{N\to\infty} \frac{N}{N + \tau_{max}} \cong \frac{N}{N} = 1.$$

Therefore, the Combination Code reaches asymptotically a full-rate and thus achieves a higher rate than the Naive Scheme. In the next session, we also show that CC gives better error rate performance than the NS and the AA.

B. Rank Criterion

Without loss of generality, some assumptions are taken to make the presentation of the derivations below clearer:

- The relative delay Δ is smaller than the code length $\frac{N}{2}$.
- We consider that $\tau_2 > \tau_1$ and hence the relative delay Δ between the two relays is positive: $\Delta = \tau_2 - \tau_1 > 0$.
- The destination is considered to be synchronous with relay R_1 so that $\tau_1 = 0$.

Let $\mathbf{X}$ be the transmitted codeword, and $\mathbf{T}$ be the erroneously decoded codeword. At the destination, the received signal $\mathbf{y}$ can be written as:

$$\mathbf{y} = \mathbf{H}\,\mathbf{X} + \mathbf{n},$$

where

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{\frac{N}{2}+\Delta} \end{bmatrix}; \qquad \mathbf{H} = \begin{bmatrix} h_1 & h_2 \end{bmatrix};$$

$$\mathbf{X} = \begin{bmatrix} X_1 & X_2 & \dots & X_{\frac{N}{2}} & \mathbf{0}^{\Delta} \\ \mathbf{0}^{\Delta} & X_1' & X_2' & \dots & X_{\frac{N}{2}}' \end{bmatrix};$$

$$\mathbf{n} = \begin{bmatrix} n_1 & n_2 & \cdots & n_{\frac{N}{2}+\Delta} \end{bmatrix}.$$

Assuming a Maximum Likelihood (ML) detection, the pairwise error probability $\mathbb{P}(\mathbf{X} \to \mathbf{T})$ can be upper bounded by the exponential bound [12]:

$$\mathbb{P}(\mathbf{X} \to \mathbf{T}) \leq \exp\left(-\frac{\mathbb{E}_{\mathbf{H}}\left(\|\mathbf{H}.(\mathbf{X}-\mathbf{T})\|^2\right)}{8N_0}\right), \qquad (1)$$

where $\mathbb{E}_{\mathbf{H}}$ is the expectation over $\mathbf{H}$.

We have $\|\mathbf{H}.(\mathbf{X}-\mathbf{T})\|^2 = \mathbf{H}.(\mathbf{X}-\mathbf{T}).(\mathbf{X}-\mathbf{T})^H.\mathbf{H}^H$, and $\mathbf{X}-\mathbf{T} = \begin{bmatrix} e_1 & e_2 & \dots & e_{\frac{N}{2}} & \mathbf{0}^{\Delta} \\ \mathbf{0}^{\Delta} & e_1' & e_2' & \dots & e_{\frac{N}{2}}' \end{bmatrix}$ is the codeword difference matrix, with $e_i = X_i - T_i$, $e_i' = X_i' - T_i'$; $\left(i = 1, \dots, \frac{N}{2}\right)$.

Let $\mathbf{A}$ be the matrix:

$$\mathbf{A} \triangleq (\mathbf{X}-\mathbf{T}).(\mathbf{X}-\mathbf{T})^H =$$

$$\begin{bmatrix} |e_1|^2 + \dots + |e_{\frac{N}{2}}|^2 & e_1'^*.e_{\Delta+1} + \dots + e_{\frac{N}{2}-\Delta}'^*.e_{\frac{N}{2}} \\ e_1'.e_{\Delta+1}^* + \dots + e_{\frac{N}{2}-\Delta}'.e_{\frac{N}{2}}^* & |e_1'|^2 + \dots + |e_{\frac{N}{2}}'|^2 \end{bmatrix}.$$

We have:

$$\det(\mathbf{A}) = \sum_{i=1}^{\frac{N}{2}} \left(|e_i|^2 \, |e_i'|^2 \right) + \sum_{\substack{i=1:\, i \neq j,\\ i \neq j+\Delta}}^{\frac{N}{2}} \left(\sum_{j=1}^{\frac{N}{2}} \left(|e_i|^2 \, |e_j'|^2 \right) \right)$$
$$- \sum_{i=1:\, i \neq j}^{\frac{N}{2}-\Delta} \left(\sum_{j=1}^{\frac{N}{2}-\Delta} \left(e_i' . e_{i+\Delta}^* . e_j'^* . e_{j+\Delta} \right) \right). \tag{2}$$

By manipulating the terms of Equation (2) conveniently, the determinant can be written as the sum of two groups of terms:

- The terms in the first group have the following form: $|e_i'|^2 \, |e_{j+\Delta}|^2 + |e_j'|^2 \, |e_{i+\Delta}|^2 - 2\,\mathcal{R}\left(e_i' . e_{i+\Delta}^* . e_j'^* . e_{j+\Delta} \right)$, with i different from j. $\mathcal{R}(x)$ being the real part of the complex number x.
- The second group terms are products of two squared modulus: $|e_i|^2 \, |e_j'|^2$; i can be equal or different from j.

The terms in the two groups are positive and so $\det(\mathbf{A})$ is a sum of positive terms that cannot be all null at the same time except for the case when the two codewords $\mathbf{X}$ and $\mathbf{T}$ are identical: $e_i = 0$ and $e_i' = 0$ for $i = 1, \ldots, \frac{N}{2}$; but this cannot happen because $\mathbf{X}$ and $\mathbf{T}$ should be two different codewords.

$\mathbf{A}$ being a Hermitian matrix, we can find a unitary matrix $\mathbf{V}$ and a positive real diagonal matrix $\mathbf{D}$ with $\mathbf{A} = \mathbf{V}^H \mathbf{D} \mathbf{V}$:

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix};$$

where λ_1 and λ_2 are the eigenvalues of matrix $\mathbf{A}$ and they are non-zero because $\det(\mathbf{A}) \neq 0$ and $\mathbf{A}$ is full rank.

Using the decomposition above, we obtain

$$\mathbf{H}.(\mathbf{X}-\mathbf{T}).(\mathbf{X}-\mathbf{T})^H.\mathbf{H}^H = \mathbf{H}.\mathbf{A}.\mathbf{H}^H = \lambda_1.|h_1.v_{11} + h_2.v_{21}|^2 + \lambda_2.|h_1.v_{12} + h_2.v_{22}|^2.$$

Therefore we have:

$$\mathbb{E}_{\mathbf{H}}\left(\|\mathbf{H}.(\mathbf{X}-\mathbf{T})\|^2 \right) = \mathbb{E}_{\mathbf{H}}\left(\sum_{i=1}^{2} \left(\lambda_i |h_1.v_{1i} + h_2.v_{2i}|^2 \right) \right) = \sum_{i=1}^{2} \left(\lambda_i \, \mathbb{E}_{\mathbf{H}}\left(|h_1.v_{1i} + h_2.v_{2i}|^2 \right) \right) = \sum_{i=1}^{2} \lambda_i \,, \tag{3}$$

because h_1 and h_2 are Gaussian variables with zero mean and a variance of $\frac{1}{2}$ for the real dimension, $(h_1.v_{1i} + h_2.v_{2i})$ are also Gaussian variables with zero mean and a variance of $\frac{1}{2}$ for the real dimension and $\mathbb{E}_{\mathbf{H}}\left(|h_1.v_{1i} + h_2.v_{2i}|^2 \right) = 1$.

Substituting (3) into (1), we obtain:

$$\mathbb{P}(\mathbf{X} \rightarrow \mathbf{T}) \leq \exp\left(-\frac{1}{8N_0} \sum_{i=1}^{2} \lambda_i \right) \leq \frac{1}{\prod_{i=1}^{2} \left(1 + \frac{\lambda_i}{8N_0} \right)} \leq \left(\prod_{i=1}^{2} \lambda_i \right)^{-1} \left(\frac{1}{8N_0} \right)^{-2}. \tag{4}$$

In Equation (4), the term $\frac{1}{8N_0}$ represents the Signal to Noise Ratio (SNR) of the network. The diversity order d can be deduced from the PEP as the exponent of the SNR [12]. Thus, we can deduce that the diversity order of the network is equal to two for any non-zero relative delay and any code length.

C. Determinant Criterion

To satisfy the determinant criterion, the minimum value of the determinant of matrix $\mathbf{A}$ over all pairs of different codewords $\mathbf{X}$ and $\mathbf{T}$ should be as large as possible.

The minimum value of $\det(\mathbf{A})$ is reached when Eq. (2) contains the minimum number of non-zero terms because it is a sum of positive terms; thus the two codewords $\mathbf{X}$ and $\mathbf{T}$ should differ in only one symbol X_m and so $e_m \neq 0$ and $e_m' \neq 0$ for one value of m (with $m \in \{1, \frac{N}{2}\}$).
Therefore, the minimum value of the determinant of $\mathbf{A}$ is:

$$\min\det(\mathbf{A}) = |e_m|^2.|e_m'|^2 \,. \tag{5}$$

The maximization of (5) will define the choice of the optimal angle α of $\theta = e^{\imath\alpha}$. In Figure 3, we plot the values of the minimal determinant (5) in function of α for symbols S_i belonging to 4-QAM and 16-QAM constellations. For 4-QAM constellation, the maximum of (5) is reached for values of α in the interval $[30°, 60°] \equiv [\frac{\pi}{6}, \frac{\pi}{3}]$. On the other hand, the maximum of (5) for 16-QAM constellation is achieved for the following values of α: $30°(\frac{\pi}{6})$, $45°(\frac{\pi}{4})$ and $60°(\frac{\pi}{3})$.
We can also obtain, by mathematical derivation, the same optimal values of α. Indeed, Eq. (5) can be written as $|Z_{2m-1} - \theta^2 Z_{2m}|^2$ with Z_{2m-1} and Z_{2m} being the differences between the S symbols of the two codewords $\mathbf{X}$ and $\mathbf{T}$ at positions $(2m-1)$ and $(2m)$ respectively. To have the optimal α, we need to calculate the values of θ that maximize $|Z_{2m-1} - \theta^2 Z_{2m}|^2$ for all possible pairs of Z_{2m-1} and Z_{2m}.

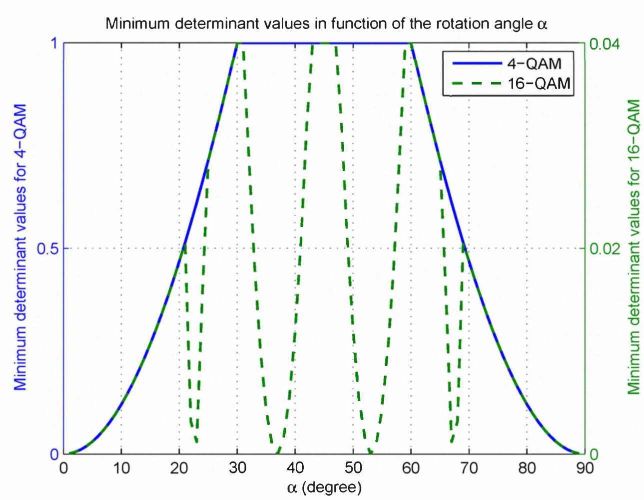

Fig. 3. Minimum determinant values in function of the rotation angle α for 4-QAM and 16-QAM symbols S_i.

IV. Code Examples and Numerical Results

Let us consider some examples of the new designed code and compare their performance to other delay tolerant schemes presented above in this paper.

For instance, we have the following matrices for the Combination Code (CC) and the Naive Scheme (NS) for $\Delta = 1$:

$$\mathbf{CC_1}=\begin{bmatrix} X_1 & X_2 & X_3 & X_4 & 0 \\ 0 & X_1' & X_2' & X_3' & X_4' \end{bmatrix};\ \mathbf{NS}=\begin{bmatrix} S_1' & S_2' & S_3' & S_4' & 0 \\ 0 & S_1' & S_2' & S_3' & S_4' \end{bmatrix}.$$

We have $X_i = S_{2i-1} + \theta S_{2i}$ and $X_i' = S_{2i-1} - \theta S_{2i}$ with $\theta = e^{\imath\frac{\pi}{4}}$ and $i = 1,\ldots,4$. For the CC, a total number of $N = 8$ symbols S_j that belong to a 4-QAM constellation is used. And for the NS, $S_i'(i = 1,\ldots,4)$ belongs to a 16-QAM constellation to have the same spectral efficiency.

In Figure 4, we simulate the Frame Error Rate (FER), the Symbol Error Rate (SER) and the Bit Error Rate (BER) for NS and CC with $\Delta = 1$ and by using an exhaustive Maximum Likelihood (ML) detector. We can see clearly that CC gives better error rate performance than NS.

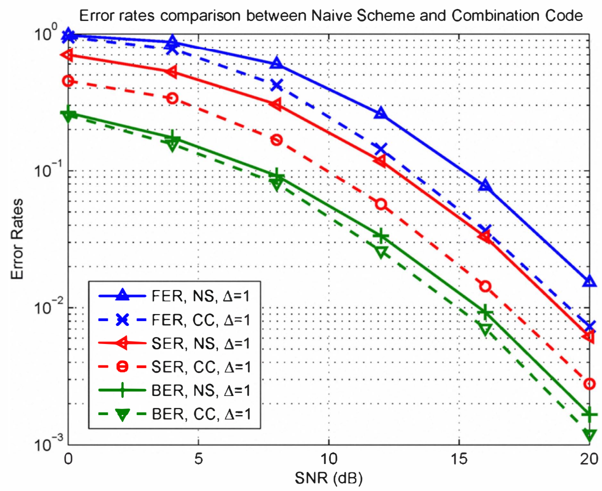

Fig. 4. Error rates of NS and CC for $N = 8$ and $\Delta = 1$.

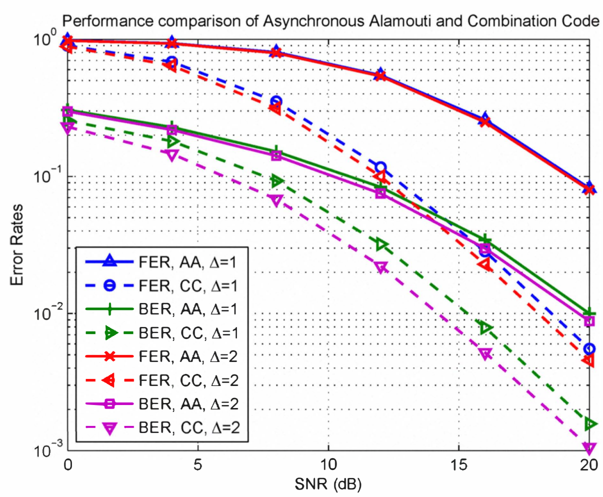

Fig. 5. Error rates of AA and CC for $\Delta = 1, 2$.

Next, we compare the performance of the CC with the delay tolerant version of the Alamouti code (AA). The two codes have the following matrices:

$$\mathbf{CC_2}=\begin{bmatrix} X_1 & X_2 & X_3 & \mathbf{0}^{\Delta} \\ \mathbf{0}^{\Delta} & X_1' & X_2' & X_3' \end{bmatrix};\ \mathbf{AA}=\begin{bmatrix} x_1 & -x_2^* & -x_2^* & \mathbf{0}^{\Delta} \\ \mathbf{0}^{\Delta} & x_2 & x_1^* & x_1^* \end{bmatrix}.$$

Here, $X_i = S_{2i-1}+\theta S_{2i}$ and $X_i' = S_{2i-1}-\theta S_{2i}$ with $\theta = e^{\imath\frac{\pi}{4}}$ and $i = 1,\ldots,3$.

To have the same spectral efficiency, $S_j(j = 1,\ldots,N = 6)$ belongs to a 4-QAM constellation and $x_i(i = 1,2)$ belongs to a 64-QAM constellation.

In Figure 5, we plot the FER and BER for two values of the relative delay $\Delta = 1, 2$ with exhaustive ML detection. We can notice that the new code designed in this paper gives better error rate performance than the Asynchronous Alamouti and that the difference in performance becomes greater when the relative delay Δ increases.

V. Conclusions

In this paper, we proposed a new coding scheme for asynchronous relay networks with single antenna nodes. The new code is based on constellation rotation and symbol combination. It was shown theoretically that it ensures a full diversity and a high rate. Examples of the code construction and comparison of performance with other codes were also given. In future work, we will propose a new coding scheme that gives a full diversity for synchronous and asynchronous relay networks.

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