

# Efficient coding/decoding scheme for phase-shift keying optical systems with differential encoding

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**Abstract:** Phase-shift keying (PSK) has been demonstrated as an efficient modulation format for long-haul high-bit-rate optical transmission systems. Differential encoding between successive PSK symbols is required for practical implementations in both coherent and direct detection. However, this kind of encoding increases the input bit error rate of the forward error correction (FEC) and thus affects its performance. In order to reduce the penalties induced by differential encoding, the authors, in this study, propose a structured symbol interleaving of the FEC codewords and the corresponding decoding algorithm. The authors' scheme allows obtaining a decoding complexity reduction of 50% compared to the usual decoding method and a redundancy reduction about 25% without any loss of performance.

## 1 Introduction

Phase-shift keying (PSK) is a very promising modulation format for high-bit-rate optical transmission systems. It has been shown that differential binary phase-shift keying (DBPSK or simply called DPSK) improves the optical signal-to-noise ratio (OSNR) by 3 dB and thus dramatically increases the transmission distance compared to on-off keying (OOK) systems [1, 2]. In direct detection systems, owing to the lack of absolute phase reference, the phase of the previous modulated symbol is used as a relative phase reference. A DPSK signal is typically demodulated by a Mach-Zehnder 1-bit-delayed interferometer that can determine the phase shift between two successive symbols. Therefore the information has to be encoded on the phase shift between different states of the constellation and this is called differential encoding. Recently, PSK optical systems using coherent detection with digital signal processing (DSP) have been receiving considerable attention. Chromatic dispersion, polarisation mode dispersion, phase noise and intermediary frequency offset can be compensated at the reception using DSP algorithms. Coherent detection allows high spectral efficiency modulation formats such as quaternary-PSK (QPSK) and polarisation division multiplexing-QPSK (PolMux-QPSK) that are good candidates for future 40 and 100 Gb/s coherent systems [3, 4]. At the receiver, the phase estimation can be performed, using phase recovery algorithms such as Viterbi algorithm [5]. However, these algorithms are unable to deal with carrier recovery phase ambiguity and differential encoding is necessary to overcome this problem.

Forward error correction (FEC) techniques have been initially introduced to increase system margins of optical transmissions, that is, to combat optical impairments

such as amplified spontaneous noise (ASE) and more recently polarisation mode dispersion or non-linear effects [6, 7]. The FECs initially developed for OOK systems have been used with phase modulation systems without any adaptation to the differential encoding scheme [8, 9]. Note that differential encoding leads to higher BER [10] which affects the FEC performance. Therefore it is common to use bit interleaving to avoid bursts of errors and all FECs used in optical transmission systems already take this into account. When differential encoding is considered, classical bit interleaving efficiently corrects the penalties but not in an optimal way.

In this paper an FEC codeword construction based on a structured symbol interleaving (SSI) of two or more codewords and the corresponding decoding algorithm is proposed. This coding/decoding scheme corrects the penalties introduced by differential encoding. It also leads to an FEC decoding complexity decrease and a significant redundancy reduction. The paper focuses on QPSK format but the proposed schemes can be applied for any modulation format using differential encoding. All the ideas presented here are available for direct detection systems and coherent systems. All the results are validated by numerical simulations and, as a matter of generality, non-linear effects are not considered and simulations are performed on an additive white Gaussian noise (AWGN) channel.

In Section 2, we recall the principles of differential encoding and an analysis of error configurations is presented. Section 3 presents the proposed structured symbol interleaving scheme and its performance is compared with the classical interleaver. In Section 4, we describe the decoding scheme and analyse its complexity. Finally in Section 5, the decoding algorithm is modified to reduce the redundancy of the FEC codewords.

## 2 Differential encoding and error configurations

### 2.1 Differential encoding

With differential encoding, the information is put on the transition between two modulated symbols of a constellation. For example, with QPSK modulation, instead of encoding the information on the signal phase, Fig. 1a, the information is encoded on the phase shift between two symbols, Fig. 1b. We call ‘data symbol’ the two bits encoding a phase transition (‘00’, ‘01’) and ‘QPSK symbol’ the modulated symbol belonging to the QPSK constellation ( $\alpha_1, \alpha_2, \dots$ ).

For example, let us consider that the last emitted QPSK symbol was  $e^{3i\pi/4}$  and that the next two bits are ‘11’. The data symbol ‘11’ corresponds to a two quadrant transition which means a  $\pi$  phase shift. The following emitted QPSK symbol is:  $e^{(3i\pi/4)+i\pi} = e^{7i\pi/4} = \alpha_4$ .

A ‘transmission error’ is a wrong decision made at the reception of a QPSK symbol. Gray mapping is generally employed to minimise the number of errors. It ensures that only one bit changes between two transitions to consecutive quadrants. As most of the errors result owing to a one-quadrant shift, at high OSNR Gray mapping reduces the total number of bit errors.

### 2.2 Error configurations

The major issue of differential encoding is that there are at least twice as many errors than in case of normal encoding. Indeed, as it can be seen in Fig. 2, one transmission error on a QPSK symbol corrupts two data symbols corresponding to:

- the phase shift between the previous QPSK symbol and the wrong symbol
- the phase shift between the wrong symbol and the next QPSK symbol

Each corrupted data symbol has two bits, and so it leads to two or four erroneous bits. Let us assume that an error occurs on a QPSK symbol and that Gray mapping is applied. If the

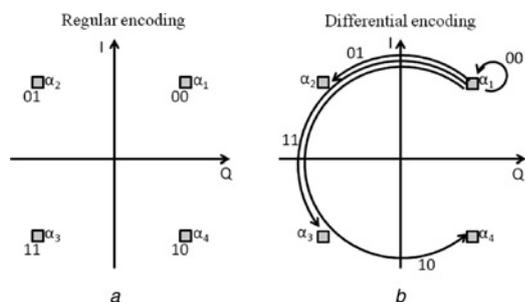


Fig. 1 Regular and differential encoding

a Regular encoding of a QPSK modulation with Grey mapping  
 b Differential encoding of a QPSK modulation with Grey mapping



Fig. 2 Error configuration on the data bits owing to a transmission error

Table 1 Errors configuration for a QPSK system with differential encoding

$n$	$Nerr_1$	$Nerr_2$	$Nerr_3$
1	+1	-1	-
	-1	+1	-
	+2	+2	-
	-2	-2	-
2	+1	+1	+2
	+2	+1	+1
	-1	-1	+2
	+2	-1	-1
	+1	+2	-1
	-1	+2	+1
	+2	0	+2
	+1	0	-1
-1	0	+1	

erroneous QPSK symbol belongs to an adjacent quadrant of the emitted symbol quadrant, there will be only two erroneous bits (one per erroneous data symbol). If the erroneous symbol belongs to the opposite quadrant of the emitted symbol quadrant, there will be four erroneous bits (two per erroneous data symbol). At high OSNR, differential encoding produces twice as many errors than classical encoding.

A transmission error corrupts two consecutive data symbols. For instance, if the first error changed the transition ‘11’ (+2 quadrants) into ‘01’ (+1 quadrant), it means that the number of quadrant of the transition has been decreased by one quadrant. As a consequence, the transition corresponding to the neighbour erroneous data symbol is increased by one quadrant from ‘00’ (0 quadrant) to ‘01’ (+1 quadrant). The first error is a -1 quadrant error and the second one is a +1 quadrant error. In general,  $n$  consecutive transmission errors affect  $n + 1$  data symbols. The corresponding errors  $Nerr_i$  expressed in quadrant, follow

$$\sum_{i=1}^{n+1} Nerr_i(\text{mod } M) = 0 \quad (1)$$

where  $M$  is the constellation size ( $M = 4$  for a QPSK constellation). Table 1 lists all the quadrant errors for one or two consecutive QPSK symbol errors. For a single transmission error ( $n = 1$ ), if one of the  $Nerr_i$  is known, the value of the second one can be directly deduced from (1). For example, if  $Nerr_2 = +1$ ,  $Nerr_1$  is equal to -1. For  $n$  consecutive transmission errors, there are some configurations in which the transmission errors can correct themselves and corrupt less than  $n + 1$  data symbols as can be seen in the last lines of Table 1.

## 3 Structured symbol interleaving

With differential encoding, the FEC receives twice as many errors and the decoding is less efficient. However, these errors are not just randomly distributed along the codewords. They come by pairs and this is very important knowledge that has to be taken into account.

One can first consider the consecutive errors as a burst of error and use a classical interleaver to overcome this problem. Randomly mixing the bits of different codewords, so that consecutive bits do not belong to the same codewords, is well-known and applied in all deployed long-haul high bit-rate optical transmission systems. However,

this technique is not optimal for the differential encoding scheme.

Here, our goal is different. The proposed construction does not try to de-correlate the consecutive bits, but, on the contrary, tries to create a certain structure between the codewords. Since a transmission error produces a pair of consecutive erroneous data symbols on a codeword, our idea is for the two corrupted data symbols to be on separated codewords. The interleaving is performed after the FEC encoding of the information. The bits of the FEC codewords are the data symbols transmitted by the modulated symbol transitions. Therefore we propose to interleave the data symbols instead of the bits. Moreover, the interleaving is not random as in the classical way but is performed alternating the data symbols from different codewords one after the other. We call this construction 'structured symbol interleaving' (SSI). For simplicity, we will present the SSI method for the case of a two-codewords interleaving. The generalisation to more codewords is easy to deduce.

Pairs of codewords are serially mixed alternating two bits (one data symbol for QPSK modulation) of each one. Then, the resulting sequence is differentially modulated into a QPSK signal, see Fig. 3. This construction ensures that two adjacent data symbols belong to two different codewords. So if an error happens during the transmission, there will be only one erroneous data symbol on each codeword instead of two on a unique codeword. The proposed scheme is optimal to combat differential encoding pairs of error.

Let  $C$  be an FEC codeword of size  $n$  with  $n-k$  redundancy bits and a  $t$  error correcting capability. The code  $C(n, k, t)$  with SSI, has the same correcting capability than the code  $C(2n, 2k, 2t)$  without interleaving. Fig. 4 presents the gain obtained by the symbol interleaving of two codewords for different families of FECs. In our simulations we have modelled the optical fibre channel by a gaussian channel like in [11]. Thus, we omit non-linear effects and assume that the equalisation is well realised. In coherent detection, the optical noise is clearly transposed in the electrical domain thus the AWGN channel model is accurate. With direct detection, ASE noise results in a  $\chi^2$  distribution but can be approximated by a Gaussian distribution for typical FEC input BER ( $10^{-1}$ – $10^{-3}$ ). The BER is plotted against the signal-to-noise ratio  $E_b/N_0$ , where  $E_b$  is the energy per transmitted information bits and  $N_0$  is the noise spectral density. Note that the SNR is proportional to the OSNR [12].

The BCH(255, 239) is a binary FEC and the coding gain obtained with the SSI is 0.5 dB at a BER =  $10^{-6}$ . Note that the performance of the BCH(255, 239,  $t=2$ ) with SSI is exactly the same as that of the BCH(511, 475,  $t=4$ ) without SSI. However, the encoding and decoding complexity of the small code is lower. The same gain of 0.5 dB is obtained for the BCH(1023, 883,  $t=14$ ) with

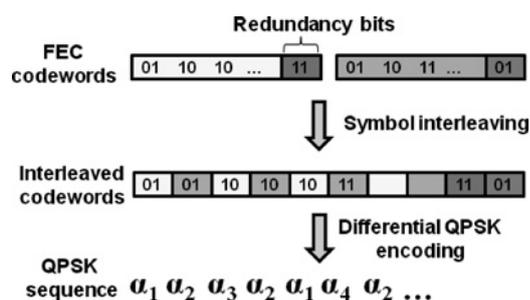


Fig. 3 Principle of symbol interleaving of two FEC codewords

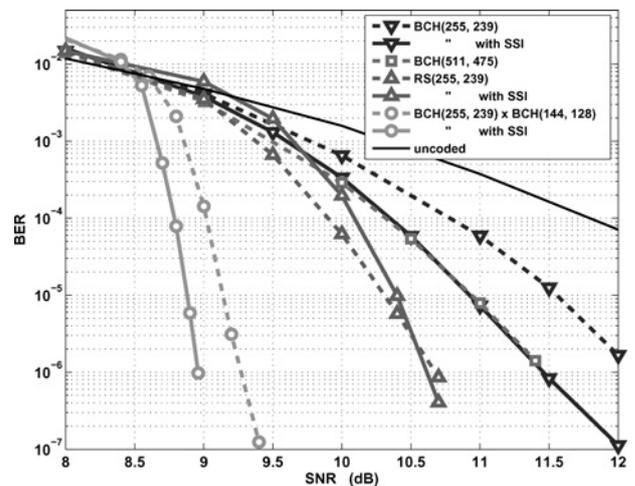


Fig. 4 Performances comparison of different FECs with and without SSI of two codewords

SSI. The product code BCH(255, 239)  $\times$  BCH(144, 128) with a hard decoding [13] is more robust to burst of errors, thanks to its construction, and therefore the coding gain is 0.2 dB. With non-binary FECs such as Reed-Solomon (RS), the efficiency of our construction is reduced. Indeed, these codes are by nature less sensitive to burst of errors and therefore less sensitive to differential encoding impairments.

The depth of an interleaver is the number of bits that are mixed. Our SSI scheme has a depth of two codewords, and so  $2n$  bits. In Fig. 5, we compare the performances of the proposed SSI scheme with the classical interleaver scheme with a 2- and 100-codewords depth ( $2n$  and  $200n$  bits). For a two-codewords depth the classical interleaver is outperformed by the SSI. In order to obtain the same performance the classical interleaving requires a large depth such as 100 codewords, which is too high and induces high implementation complexity.

We can easily conclude that the SSI is more adapted to the differential encoding than the classical interleaving. We will show in the next session that SSI also allows reducing decoding complexity. Note that the proposed construction can be applied to any kind of constellation using differential encoding. For instance, DBPSK has been demonstrated particularly well adapted for non-coherent

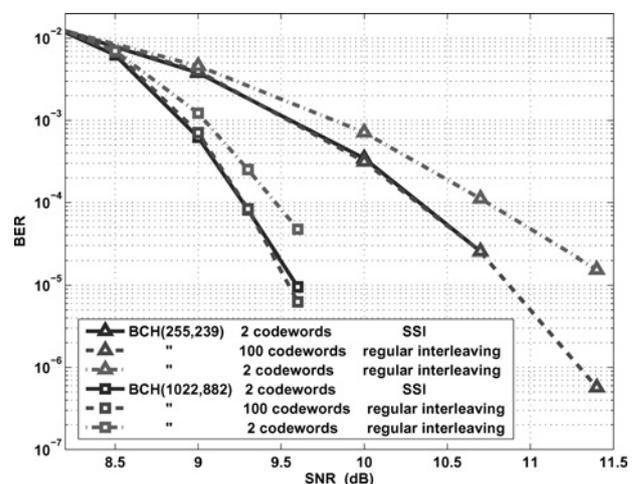


Fig. 5 Comparison between the proposed SSI and the classical random interleaver

optical ultra-long-haul transmission systems. It is a two-state constellation and each state is coded by one bit. So the corresponding construction for a DPSK scheme is a bit interleaving between two codewords.

### 4 Complexity reduction of the FEC decoding

#### 4.1 Principles of the decoding complexity reduction

We have seen that errors come in pairs of erroneous data symbols, each one belonging to different codewords (see Fig. 3). To recover the data, algebraic decoding of each codeword is performed after the de-interleaving. We call ‘algebraic decoding’ the regular way to decode FECs.

We now propose to reduce the decoding complexity by performing the algebraic decoding on only one of the two codewords and to deduce the decoding of the second one. Indeed, the algebraic decoding gives us the position and the value of the error of one of the erroneous data symbol. As errors come in pairs, we know that the other erroneous data symbol is one of its direct neighbours. Moreover, as explained in Section 2 in Table 1, if we know the error value of the first error we can deduce the correction for the second one. The remaining question is: which neighbour is the erroneous one?

For instance, in Fig. 6, the algebraic decoding has corrected an error on the first codeword (the tenth bit). The correction has changed a ‘11’ transition into a ‘10’ transition: it corresponds to a +1 quadrant correction. From Table 1, we know that the correction of the other erroneous data symbol has to be a -1 quadrant correction (under the assumption of a single transmission error). The last step is to determine if the second erroneous data symbol is the left or the right neighbour.

We can alternatively correct both of the neighbours and decide afterwards which one corresponds to the right correction. A pattern of all possible corrections for the second codeword is created. For example, in Fig. 6, the pattern has two entries: a codeword corresponding to the correction of the right neighbour and a codeword corresponding to the correction of the left neighbour. If many errors have been corrected by the algebraic decoding, all the configurations have to be checked. Therefore the size of the pattern grows exponentially with the number of corrected errors on the first codeword. The right correction is the one giving a valid codeword of the FEC. In linear block codes (RS, BCH, LDPC etc.), each codeword  $c = [c_1 c_2 c_3 \dots c_n]$  is orthogonal to the parity check matrix  $H$  of the code [14]

$$cH^T = 0 \tag{2}$$

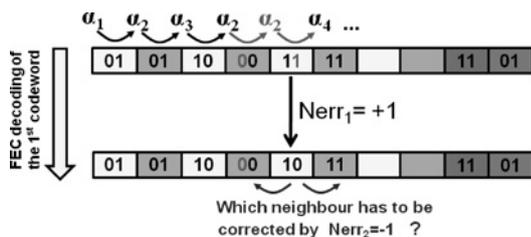


Fig. 6 Error on the first codeword is detected and corrected using algebraic decoding

Position of the error on the second codeword is one of the neighbours and the correction to apply is known

Therefore in order to recognise the right correction, we can check which one gives a valid codeword of the FEC by computing the syndrome

$$\text{synd} = cH^T \tag{3}$$

If the syndrome is null, the codeword is valid. In the pattern, only one of the two corrections can lead to a valid codeword. In one case the erroneous data symbol is corrected, whereas in the other case, the other neighbour is corrupted and both neighbours are then erroneous. If there is no valid correction in the pattern, algebraic decoding of the second codeword has to be performed. The decoding algorithm can be summarised in Fig. 7.

Note that the pattern can consider consecutive error cases; however, the size of the pattern increases. The corrections are based on the error configurations listed in Table 1 for  $n > 1$ . For example, if the first decoding corrected a single error on the  $k$ th symbol, this can come from an error on one or both of its neighbours (the  $k-1$ th and/or the  $k+1$ th symbol). Let us imagine that the correction was a +1 correction. To consider the single error configuration, we will make a -1 quadrant correction on the  $k-1$ th and then a -1 quadrant correction on the  $k+1$ th symbol. To consider the two consecutive errors configuration, we will make a +2 quadrant correction on the  $k-1$ th symbol and a +1 quadrant correction on the  $k+1$ th symbol (line 5 of Table 1) and the opposite (line 4 of Table 1)

#### 4.2 Evaluation of the decoding complexity

In our algorithm the first codeword is always decoded algebraically and the complexity reduction depends on the second codeword decoding. Syndrome computation represents the most complex operation, as it has to be repeated for all the codeword corrections of the pattern. As the size of the pattern grows exponentially with the number of detected errors, the decoding can become prohibitively complex with powerful FECs. For instance, if the FEC can detect and correct 20 errors, the pattern size may reach  $2^{20}$ .

The size of the correction pattern can be reduced by eliminating some cases. Instead of applying a correction on both neighbour data symbols, only the least reliable one is

- 1: Algebraic decoding of the first codeword
- 2: **if** the first codeword was valid **then**
- 3:     End
- 4: **else**
- 5:     Creation of a correction pattern
- 6:     **if** there is one valid codeword **then**
- 7:         End
- 8:     **else**
- 9:         Algebraic decoding of the second codeword
- 10:     **end if**
- 11: **end if**

Fig. 7 Decoding algorithm

considered. The least reliable data symbol is the one corresponding to the least reliable QPSK symbol. The reliability of the QPSK symbols can be obtained computing their log likelihood ratio (LLR) [15]

$$\text{LLR}(S) = \log \frac{P(S = \alpha_r | r)}{P(S \neq \alpha_r | r)} \quad (4)$$

where  $S$  is the transmitted QPSK symbols,  $r$  the received QPSK symbols and  $\alpha_r$  is its state in the constellation. The QPSK symbol having the smallest absolute value of LLR is the least reliable. We assume that it is the wrong one and we only correct the corresponding data symbol. Therefore the pattern is now reduced to only one codeword. There is only one syndrome computation to perform to check whether our correction is valid. The complexity of this operation can be considered negligible in comparison with an algebraic decoding. Thus, we can decode two codewords with the complexity of a single algebraic decoding.

If our correction is wrong, the algebraic decoding of the second codeword also has to be performed and no complexity gain is obtained. The complexity of our algorithm is evaluated by the average number of FEC decoding performed to decode both codewords. We define the total complexity reduction as the ratio between this number and the regular case where both codewords are always decoded.

In Fig. 8, the complexity reduction is plotted over the output BER of the FEC. A 50% decoding complexity reduction is reached with the BCH(1022, 882) code which means that the decoding of one codeword is most of the time enough to deduce the correction for the second one. Indeed, the input BER needed to achieve an error-free transmission ( $\text{BER} < 10^{-12}$ ) with this FEC is high enough to ensure that the probability of having consecutive errors and/or making a mistake on the reliability of the QPSK symbols is very small. The product code BCH(255, 239)  $\times$  BCH(144, 128) with hard decoding is a more powerful FEC (see Fig. 4) and works at lower SNR. So, it is more probable that the correction of the second codeword fails and that the algebraic decoding has to be done. As the average number of decoding for the second codeword is higher, the total reduction reaches 45%. The coding gain of

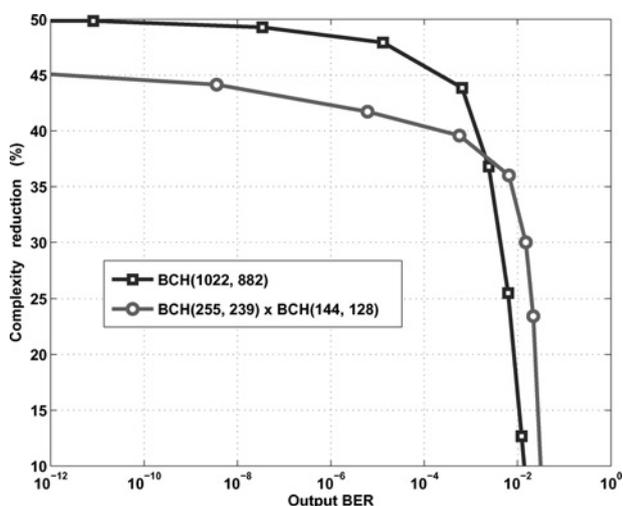


Fig. 8 Percentage of complexity reduction measured for different output BER

the SSI scheme is kept by the proposed decoding, and so we have a complexity gain without any performance loss.

## 5 Redundancy reduction

The previous section has shown how the decoding of the first codeword can lead to the decoding of the second codeword with little effort. As the error correction capability of the second codeword is not really used, it is not necessary to encode it in the same way as the first codeword.

We propose to encode the data by two different FECs before the SSI. An FEC with low correction capability, hence low redundancy, is chosen to encode the second codeword. With this configuration the total redundancy is decreased compared to the case using similar FECs. Let us consider two FECs:  $C_1(n_1, k)$  and  $C_2(n_2, k)$  with  $n_2 < n_1$ . Both FECs here use the same number of information bits  $k$ , but different constructions can be imagined to be more efficient. The resulted redundancy is

$$r = \frac{(n_1 + n_2) - 2k}{(n_1 + n_2)} \quad (5)$$

$r$  is inferior to  $1 - (k/n_1)$  (the total redundancy when the same FEC is used).

The problem of using a less efficient FEC for the second codeword arises when algebraic decoding has to be performed. Indeed, the number of errors on each codeword is the same but the second FEC is not able to correct as many errors as the first one. It means that the second decoding will often fail and this will lead to an error floor on the performance.

Algebraic decoding of the second codeword takes place when our algorithm proposes a wrong correction. However, although the correction is not valid, it is close to the right one. Most of the errors have already been removed and the algebraic decoding has only to deal with the remaining errors. Therefore a less efficient FEC can be used for the second codeword if the number of remaining errors is not too large.

In Fig. 9, we plot the performance of some combinations of FECs offering different redundancy reductions. The codes are decoded as described in the previous section. BCH(1022, 882) is used as the FEC of the first codeword. When the

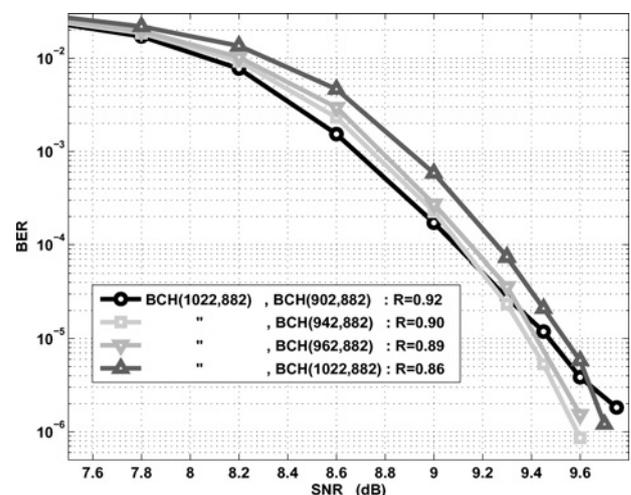


Fig. 9 Performance of various combinations of FEC with different redundancies

same FEC is used for the second codeword, the total redundancy is 14% (rate  $R = 0.86$ ). However, if we use a BCH(962, 882) or a BCH(942, 882), leading, respectively, to a total redundancy of 11% (rate  $R = 0.89$ ) and 10% (rate  $R = 0.90$ ), no performance degradation is observed until  $BER = 10^{-6}$ . The BCH(902, 882) is not able to correct all the errors left by the first step of the algorithm and its performance is decreased. For a chosen FEC combination presenting an error floor, a concatenated scheme can be employed where the inner code uses the redundancy reduction proposed technique and the outer code removes the error floor. Concatenated schemes have already been proposed to remove error floors left by LDPC code decoding and therefore the SSI coding/decoding scheme can be used to reduce the LDPC redundancy while the outer code corrects at the same time the error floor of the LDPC decoding and of the SSI decoding.

It can be observed in the figure, that FEC with small redundancies perform slightly better. Indeed, the energy per information bit is equal to  $E_b = E_s / (R \cdot k_{\text{QPSK}})$  where  $E_s$  is the energy per modulated symbol,  $k_{\text{QPSK}} = 2$  is the modulation rate and  $R$  is the FEC rate. Therefore FEC with small redundancies, high rates correspond to smaller SNR. In Fig. 9, the FEC combinations corresponding to the rates 0.86, 0.89 and 0.90 have the same correction capability on the coded bits; however, the ones with small redundancies correspond to more information bits and so perform better.

In our simulations, we observed no performance degradation using the BCH(942, 882) instead of BCH(1022, 882) as the FEC of the second codeword. Moreover, the number of redundancy bits is reduced from 280 (140 redundancy bits on each codeword) to 200 (140 on the first codeword and 60 on the second) which means that the redundancy is reduced by 29%.

## 6 Conclusions

We have proposed a coding scheme based on an SSI of two FEC codewords to mitigate the differential encoding penalties. We have obtained a significant coding gain for binary FECs. An original decoding of the SSI construction has also been proposed, working with any FEC and offering decoding complexity reduction that can reach 50% without performance degradation. Finally, we have shown that according to the SSI coding/decoding scheme we can choose an FEC with smaller redundancy. In our simulations, the rate has been increased from 0.86 to 0.9 with no penalties by choosing an FEC with 29% less redundancy bits.

A major issue of actual and future generations of high bit-rate optical transmission systems is the implementation of the new generation of FECs (low-density parity-check codes, Turbo code...). These codes have in particular, high redundancies and decoding complexity algorithms.

Therefore the proposed SSI coding/decoding scheme can be very profitable to these systems.

The future works will be focused on the experimental validation of these results.

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