PDL Mitigation in PolMux OFDM Systems using Golden and Silver Polarization-Time Codes

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Abstract: Polarization-Time coding can highly mitigate PDL impairments in optical OFDM systems. The performances of two full-rate Polarization-Time codes, the Golden and Silver codes, have been evaluated showing important coding gains.

OCIS codes: (060.2330); Fiber optics communications; (060.4510) Optical communications Modulation

1. Introduction

Polarization multiplexed (PolMux) formats are very attractive for high bit-rates optical transmission systems but can highly suffers from the polarization effect in fibers such as polarization mode dispersion (PMD) and polarization dependent loss (PDL). The polarization multiplexed transmissions can be seen as a 2×2 multiple-input multiple-output (MIMO) systems. Indeed, the signal can be multiplexed on 2 polarizations at the emission, which corresponds to the multiple inputs, and received on 2 polarizations, which corresponds to the multiple outputs. Space-Time codes have been introduced in wireless communications in order to mitigate the fading by exploiting all the degrees of freedom of the MIMO channel. They can be adapted for optical transmissions and have been referred in the literature as Polarization-Time codes [1]. Their decoding needs a linear and not dispersive channel which requires the use of optical OFDM. The Golden code [2] and the Silver code [3] are, in this order, the two best codes on 2×2 wireless MIMO channel, we propose to evaluate their performances on the optical channel. We show that Polarization-Time coding techniques are not able to manage dispersion impairments such as PMD but can very efficiently mitigate the performances degradation induced by PDL. Moreover, we also show that unlike in wireless transmission, the Silver code here performs better than the Golden code.

2. Polarization multiplexed coherent optical OFDM

We consider a PolMux coherent optical OFDM system as described in [4]. The PolMux signal is transmitted and received on two polarizations and this can be seen as a 2×2 MIMO system. The principle of OFDM format is to multiplex the information over multiple orthogonal subcarriers independently modulated. The modulation is performed in the frequency domain and converted in the time domain by an inverse FFT. In order to manage dispersion effect such as chromatic or polarization mode dispersion, a cyclic prefix is added on each OFDM symbol and thus we have for each subcarrier a linear and not dispersive channel once back to the frequency domain at the reception:

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{N}_k \tag{1}$$

where $\mathbf{H}_k$ is the transfer matrix of the channel and $\mathbf{X}_k$,$\mathbf{Y}_k$,$\mathbf{N}_k$ are respectively the transmitted symbols, the received symbols and the noise on the k^{th} subcarrier. The optimal way to decode the received symbol is to perform a maximum likelihood (ML) decoding searching the symbol which minimizes the quadratic distance with the received symbol. This decoding requires the knowledge of the transfer matrix and this can be obtained by training sequence. As the considered system is a 2×2 MIMO system and the modulation formats have small size (QPSK), ML decoding can be performed with reasonable complexity.
The capacity of this channel for each subcarrier can be expressed as in [5]:

$$C_k = \sum_{i=1}^{2} E_{\lambda_i} \left\{ \log_2 \left(1 + \frac{\rho}{2} \lambda_i^2 \right) \right\} \tag{2}$$

where ρ is the signal to noise ratio per symbol for the sub-carrier and λ_i are the singular values of $\mathbf{H}_k$. Note that when only PMD is considered, as the transfer matrix is unitary [6] (both singular values are equal to 1), the capacity is not reduced and equivalent to the capacity of two parallel Gaussian channels. Therefore Polarization-Time coding techniques can't bring any improvement to a system with only PMD impairements.

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We now consider a transmission affected by PDL and the transfer matrix of the channel is modeled as in [7]:

$$\mathbf{H_{PDL}} = \mathbf{R}_\alpha \begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & \sqrt{1+\gamma} \end{bmatrix} \mathbf{R}_\beta \tag{3}$$

$\mathbf{R}_\alpha$ and $\mathbf{R}_\beta$ are random rotation matrices and the PDL is expressed as $\mathbf{\Gamma}_{dB} = 10\log_{10}(1-\gamma)/(1+\gamma)$.

3. Polarization-Time coding

OFDM format ensures a linear and not dispersive channel and thus, Polarization-Time coding can be applied as in Fig. 1.

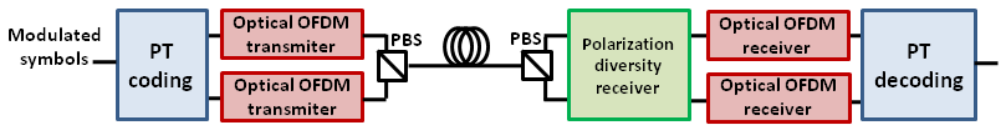

Fig. 1. Scheme of a 2x2 OFDM transmission with Polarization-Time coding

The principle of Polarization-Time codes is to send a linear combination of modulated symbols on each polarization (pol_1, pol_2) during several symbol times. On a 2×2 MIMO channel, the maximum achievable rate (or full rate), corresponds to 2 transmitted symbols per symbol time (i.e 2 symb/channel use). Alamouti code has often been proposed for optical systems [1] but it can only reach a rate of 1 symb/cu which can be seen as a 50% symbol overhead. Polarization-Time coding enhance the performance bringing coding and diversity gain. In wireless communications, the code offering the highest coding gain for a 2×2 MIMO system is the Golden code [2], followed by the Silver code which has the second highest coding gain [3]. The transmitted symbols are given by a codeword matrix which is equal to:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{pol_1,T_1} & \mathbf{X}_{pol_1,T_2} \\ \mathbf{X}_{pol_2,T_1} & \mathbf{X}_{pol_2,T_2} \end{bmatrix} = \begin{cases} \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\,(S_1+\theta S_2) & \alpha\,(S_3+\theta S_4) \\ i\overline{\alpha}\,(S_3+\overline{\theta} S_4) & \overline{\alpha}\,(S_1+\overline{\theta} S_2) \end{bmatrix} & \text{Golden code} \\ \\ \begin{bmatrix} S_1+Z_3 & -S_2^*-Z_4^* \\ S_2-Z_4 & S_1^*-Z_3^* \end{bmatrix} & \text{Silver code} \end{cases} \tag{4}$$

where $Z_3=\frac{1}{7}\left[(S_3-S_4)+i\,(S_3+2S_4)\right]$, $Z_4=\frac{1}{7}\left[(S_3+S_4)+i\,(2S_3-S_4)\right]$, $\theta=\frac{1+\sqrt{5}}{2}$, $\overline{\theta}=\frac{1-\sqrt{5}}{2}$, $\alpha=1+i+i\theta$ and $\overline{\alpha}=1+i+i\overline{\theta}$. Both are achieving the full rate of 2 symb/cu as 4 modulated symbols (S_1, S_2, S_3, S_4) are transmitted during 2 symbol times (T_1,T_2).

On Rayleigh fading channel the coding gain is proportional to the minimum determinant of the difference of two codeword matrices. It is equal to $\frac{1}{7}$ for the Silver and $\frac{1}{5}$ for the Golden and thus, Golden code outperforms the Silver code. However the optical fiber channel is different from the wireless channel and the performances of the Polarization-Time codes are not based on the same criteria. The error probability is function of the Euclidian distance between each pair of codewords and we call d_{min} the minimum distance. The transfer matrix $\mathbf{H}$ modifies the codeword constellation and reduces the minimum distance between the codewords, which increases the error probability. Therefore the minimum distance becomes:

$$d_{min} = \min_{\mathbf{X}_1,\mathbf{X}_2 \in \mathcal{C}} \left(\|\mathbf{H}\,(\mathbf{X}_1-\mathbf{X}_2)\| \right) \tag{5}$$

The PT code having the best performances under the optical channel is the one having the highest minimum distance d_{min}.

4. Simulation and results

We consider PolMux OFDM system leading to the channel model of Eq. 1. The transfer matrix is modeled by Eq. 3 and the modulated symbols belong to a QPSK constellation. $\mathbf{H}$ is function of the random angles α and β but the rotation matrix R_α has no effect on d_{min}. Fig. 2 shows the minimum distance of the Silver and Golden codes for different angles β. d_{min} shows a $\frac{\pi}{2}$ periodicity. As the rotation matrix angle is uniformly distributed over $[0, 2\pi]$, we can deduce the average minimum distance $E_{\mathbf{H}}\left[d_{min}\right]$ of each code on the optical channel by averaging d_{min} on $\left[0, \frac{\pi}{2}\right]$.

On Fig. 3 we can notice that the Silver code always has a higher average minimum distance than the Golden code for the different values of PDL. Note that the average minimum distance of the Silver code starts to decrease only

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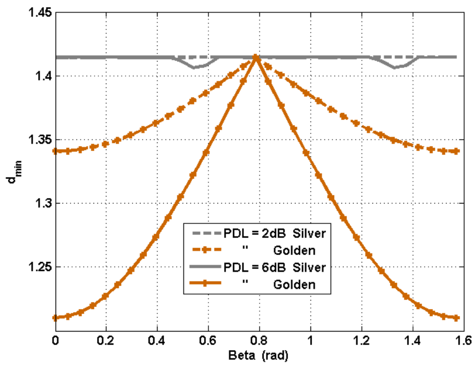

Fig. 2. Minimum distance of Polarization-Time codes depending on the rotation angle β of the transfer matrix for a QPSK constellation

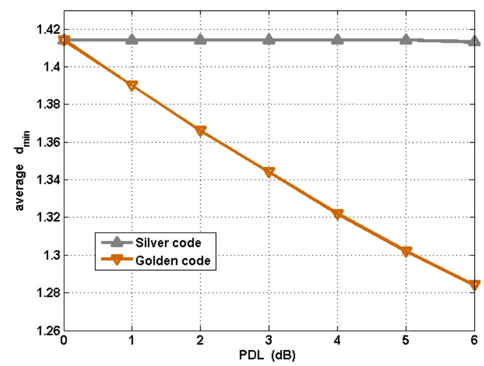

Fig. 3. Average minimum distance of Polarization-Time codes function of PDL for a QPSK constellation

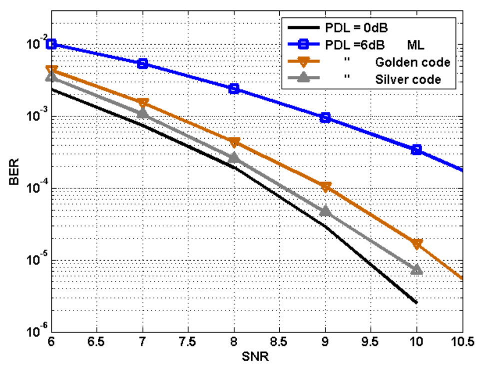

Fig. 4. Performances with and without Polarization-Time coding

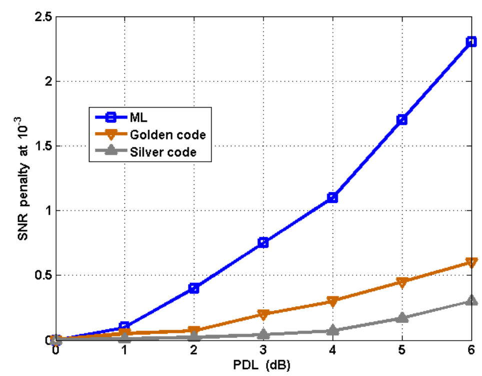

Fig. 5. SNR penalties introduced by PDL at $\mathrm{BER} = 10^{-3}$ with and without Polarization-Time coding

for PDL values $> 6\mathrm{dB}$. We can clearly see the important penalty reduction brought by Polarization-Time coding on Fig. 4. The performances of the Polarization-Time codes are very close to the case without PDL. The SNR penalties at $\mathrm{BER} = 10^{-3}$ (the expected FEC threshold) are computed for different PDL values on Fig. 5. With a $\mathrm{PDL} = 6\mathrm{dB}$, the penalties with the Golden and the Silver are reduced from $2.3\mathrm{dB}$ to respectively $0.6\mathrm{dB}$ and $0.3\mathrm{dB}$. This corresponds to an approximate coding gain of $2\mathrm{dB}$. Moreover, The Silver code is indeed outperforming the Golden code. When there is no PDL, those two Polarization-Time codes do not introduce any penalties as they have by construction a full rate so, the use of a Polarization-Time code is always profitable.

5. Conclusion

We have introduced the Golden and the Silver Polarization-Time codes in order to mitigate the PDL impairments in 2×2 coherent optical PolMux OFDM systems. Both codes do not introduce any symbol overhead and have good performances. Polarization-Time coding is useless against PMD but can very efficiently mitigate the PDL impairments. Indeed, important coding gains have been observed. Moreover, we have shown that unlike in wireless communication, the Golden code is not the best code anymore and the Silver code is a better solution against PDL.

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