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Space-Time codes for optical fiber communication with polarization multiplexing

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***Abstract*—Polarization effects may induce severe performances degradation in polarization multiplexed optical fiber transmissions. Those systems can be seen as 2×2 multi-antennas systems as the emitted polarizations can be considered as 2 input signals and the received polarizations as 2 output signals. Therefore, Space-Time code can be used to take benefit of this configuration and enhance the transmission performances but they have to be combined with optical OFDM to suppress the fiber dispersion and allow their decoding. In wireless 2×2 multi-antennas systems, the Golden and the Silver code are respectively the two best Space-Time codes so, we propose to use those two codes on polarization multiplexed systems. The performances of the Space-Time codes on the optical fiber channel are different than on the wireless channel. Simulations show than the Silver code outperforms the Golden code. Nevertheless, we also show that Space-Time coding can dramatically mitigate the polarization dependent loss (PDL) impairments.**

I. Introduction

Polarization multiplexed (PolMux) formats have become very attractive for 40 Gb/s and 100 Gb/s optical transmission systems. Indeed the spectral efficiency can be doubled emitting the data simultaneously on two orthogonal polarizations. PolMux transmissions require efficient digital signal processing in order to recover the polarizations which have been affected by the birefringence of the fiber and several transmission impairments. Polarization Mode Dispersion (PMD) introduces a frequency dependent phase shift to the signal and corresponds to a major source of degradation at high bit-rates [1]. It can be mitigated by efficient equalization techniques [2] or advanced modulation formats such as OFDM [3]. Polarization Dependent Loss (PDL) is another source of impairments introduced by all the in-line optical elements of the systems (isolator, amplifier…) which attenuates differently the two polarization components of the signal. PDL induces polarization dependent optical power fluctuations resulting in unequal optical signal-to-noise ratio (OSNR) on the two PolMux signals. Note that PDL can't be efficiently mitigated by digital equalization techniques.

The polarization multiplexed transmissions can be seen as a 2×2 multiple-input multiple-output (MIMO) systems. Indeed, the signal can be multiplexed on 2 polarizations at the emission, which corresponds to the multiple inputs, and received on 2 polarizations, which corresponds to the multiple outputs. Space-Time codes have been introduced in wireless communications to exploit all the degrees of freedom of the MIMO channel. They can be adapted for optical transmissions and have been referred in the literature [8] as Polarization-Time codes. However, their decoding requires a linear and not dispersive channel which is not the case of the optical fiber. Optical OFDM can manage dispersion impairments considering the channel in the frequency domain and therefore has to be combined with the Polarization-Time coding techniques. The Golden code [10] and the Silver code [11] are, in this order, the two best codes on 2×2 Rayleigh fading channel and we propose to evaluate the performances of those two codes on a optical channel affected by PDL. We show that Polarization-Times coding techniques are not able to manage dispersion impairments such as PMD but can very efficiently mitigate the PDL performances degradation. Moreover, unlike in wireless transmission, the Silver code here performs better than the Golden code.

After presenting the considered optical fiber channel model and describing the optical OFDM format, the Polarization-Time codes will be introduced. We give an overview of the Polarization-Time coding scheme in the literature and show that they are sub-optimal. Finally, performance evaluation through simulation results will be proposed.

II. Optical channel model

In multi-carrier high bit-rates optical transmissions, the transmitted signal is affected by many kinds of impairments in the fiber such as: chromatic dispersion, polarization mode dispersion, polarization dependent loss and cross phase modulation (XPM). In this paper, we will focus our attention here, on the PMD and the PDL which are linear effects and can be modeled using a transfer matrix approach. The transfer matrix $\mathbf{H}(\omega)$ of the fiber is called Jones matrix; it is a 2×2 complex matrix expressing the relation, between the two orthogonal components of the electric field in the frequency domain, at the input and the output of the fiber:

$$\begin{bmatrix} E_x^{out} \\ E_y^{out} \end{bmatrix} = \mathbf{H}(\omega) \cdot \begin{bmatrix} E_x^{in} \\ E_y^{in} \end{bmatrix} \quad (1)$$

A. Polarization mode dispersion

Polarization mode dispersion introduces differential group delay (DGD) between the two orthogonal principal states of polarization (PSP). Indeed, due to local constraints in the fiber, the two components of the electrical field do not travel at the same velocity. As the local birefringence is a stochastic process, the DGD varies in time. In the first order PMD approximation, DGD can be considered constant over the spectrum, however, with the increase of the bit-rates, this approximation does not stand anymore and the DGD has to be considered wavelength dependent. The PMD is often modeled by a transfer matrix being a concatenation of random rotation matrix and wavelength dependent birefringent matrices [5]:

$$\mathbf{H_{PMD}}(\omega) = \prod_{m=1}^{N} R_m DGD_m \tag{2}$$

$$DGD_m = \begin{bmatrix} \exp\left(i\frac{\omega}{2}dgd(\omega)\right) & 0 \\ 0 & \exp\left(-i\frac{\omega}{2}dgd(\omega)\right) \end{bmatrix}$$

The rotation matrices R_m depend on two random angles θ_m and φ_m describing the local orientation mismatch between the PSP of the fiber and the polarizations of the signal, dgd is the DGD of the section.

B. Polarization dependent loss

Optical transmission systems have become very complex and include a large number of inline optical components having significant PDL such as isolators, couplers or amplifiers. In PolMux systems, PDL induces a fluctuation of the polarization components power level during the transmission changing the OSNR and leading to BER degradation at the receiver. The optical elements having PDL in the system can be considered as elements attenuating the signal more along one of the PSP than the other one. The attenuation of each polarization depends on their orientations compare to the axes of the optical element. We consider the model used in [6] to emulate the PDL:

$$\mathbf{H_{PDL}} = \mathbf{R}_\alpha \begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & \sqrt{1+\gamma} \end{bmatrix} \mathbf{R}_\beta \tag{3}$$

$\mathbf{R}_\alpha$ and $\mathbf{R}_\beta$ are random rotation matrices representing the orientation mismatch between the polarization states and the axes of the optical component having PDL. The PDL is defined by:

$$\mathbf{\Gamma}_{dB} = 10\log_{10}\frac{1-\gamma}{1+\gamma} \tag{4}$$

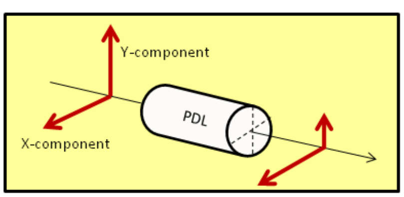

Fig. 1. Principle of PDL when the axes of the optical element are parallel to the polarizations of the input signal

III. Optical OFDM

OFDM has been recently investigated in optical transmission systems because of its high potential of dispersion mitigation. Direct detection and coherent OFDM have been introduced showing excellent behavior against chromatic dispersion and PMD [3][4].

A. OFDM Channel model

The principle of OFDM format is to multiplex the information over multiple orthogonal subcarriers independently modulated. The modulation is realized in the frequency domain and converted in the time domain by an inverse FFT. The advantage is that the channel is not dispersive in the frequency domain and it can highly simplify the equalization. Indeed, due to the impairments of the fiber such as chromatic dispersion or PMD, there might be Inter-Symbol Interferences (ISI). In order to recover the not dispersive channel, ISI has to be removed. The solution offered by the OFDM format is to add a cyclic prefix at the beginning of each OFDM symbol. Thanks to this cyclic structure, the ISI will be transformed, once converted back in the frequency domain, into a scalar multiplication. A detailed scheme of an optical OFDM transmission can be found in [3]. In the following, we will assume an OFDM transmission in which the cyclic prefix is well dimensioned to ensure a not dispersive channel. The received modulated symbols can be expressed as:

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{X}_k + \mathbf{N}_k \tag{5}$$

where $\mathbf{H}_k$ is the transfer matrix of the channel for the k^{th} subcarrier and $\mathbf{X}_k$,$\mathbf{Y}_k$,$\mathbf{N}_k$ are respectively the transmitted symbols, the received symbols and the noise on the k^{th} subcarrier. As described in [4], optical OFDM can support PolMux formats and this can be seen as a MIMO not dispersive linear channel where the polarizations have the role of the antennas in wireless communication.

B. Capacity of the OFDM channel

The capacity of an $n_t \times n_r$ MIMO channel (using n_t antennas or polarization, at the emission and n_r antennas or polarization, at the reception) is equal to [14]:

$$C_k = E_H\left\{\log_2\left(\det\left[\mathbf{I} + \frac{\rho}{n_t}\mathbf{H}_k\mathbf{H}_k^\dagger\right]\right)\right\} \tag{6}$$

After a singular value decomposition of the matrix $\mathbf{H}_k$, it leads to:

$$C_k = \sum_{i=1}^{\min(n_t,n_r)} E_H\left\{\log_2\left(1 + \frac{\rho}{n_t}\lambda_i^2\right)\right\} \tag{7}$$

where ρ is the signal to noise ratio per symbol on the subcarrier and the λ_i are its singular values of $\mathbf{H}_k$. A MIMO channel can be seen as $\min(n_t,n_r)$ parallel AWGN channels having their own capacity. The OFDM is a multi-carrier format and the total capacity of the transmission is the sum of the capacity of all sub-carriers. From now on, the index corresponding to the subcarriers will be dropped for simplicity.

C. Decoding linear channel

For each subcarrier we have the channel model of Eq.1. The transfer matrix can be estimated and will be supposed known at the receiver. The optimal way to recover the information is to realize a maximum likelihood (ML) decoding. The principle is to find the estimated symbol vector minimizing the quadratic distance with the received vector:

$$\widehat{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2 \quad (8)$$

The ML decoding can be performed for example, by an exhaustive research over all the constellation points. In optical transmission systems, BPSK and QPSK modulation are commonly used and doing a ML decoding by an exhaustive research on such small size formats is possible and the induced complexity is reasonable.

IV. Polarization-Time coding

A. Space-Time codes in optical transmission systems

In optical systems the MIMO channel is brought by the polarization thus, the Space-Time codes are referred as Polarization-Time codes. The optical systems can use up to two polarizations at the emission ($n_t \leq 2$) and two polarizations at the reception ($n_r \leq 2$).We will consider the case where both polarizations are used at the transmitter and at the receiver thus, we have a 2×2 MIMO channel. The Space-Time decoding can only be performed under the assumption of not dispersive channel and $\mathbf{H}$ constant during the codeword time duration. In optical transmission the channel is changing slowly in comparison to the bit-time and so can be considered constant. Optical OFDM ensures a not dispersive model of the channel in the frequency domain.

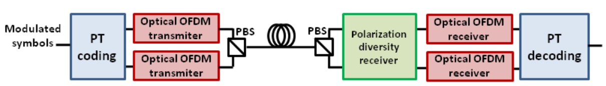

Fig. 2. Scheme of a 2x2 OFDM transmission with Polarization-Time coding

Systems combining OFDM format and Polarization-Time coding have already been proposed in the literature. In [4], a good description of all the PolMux OFDM configurations is given. For the 2×1 and the 2×2 configurations, a polarization diversity transmitter with a symbol rate of 0.5 symbol/cu (symbol per channel use) has been presented. Therefore, this scheme introduces an important redundancy, as a 2×2 MIMO systems can have a maximum rate of 2 symbol/cu. In [9], Alamouti code has been proposed for a 2×2 PolMux OFDM system, as described on Fig.2. A method combining PT decoding and carrier recovery is also presented. However, Alamouti code has a rate of 1 symbol/cu corresponding to a 50% redundancy. Moreover, neither coding gain nor diversity recovery was observed. The authors have only considered PMD with a transfer matrix of the fiber having the form of Eq.2. Although PMD can be a very limiting impairment for transmission, it does not reduce the capacity of the channel. Using Eq.6 and noticing that the transfer matrix of the fiber is always unitary (both singular values are equal to 1), the capacity of the 2×2 MIMO channel with only PMD can be expressed as:

$$C = 2\log_2 \frac{\rho}{2} \quad (9)$$

This is capacity correspond to two parallel AWGN channels. PMD can be considered as information mixing between the two channels but there is no loss of information. Therefore, Polarization-Time coding doesn't bring any gain.

B. The Golden code and the Silver code

The principle of Space-Time codes, here used as Polarization-Time codes, is to send a combination of different modulated symbols on each polarization during several symbol times (i.e channel use). On a 2×2 MIMO channel, to achieve full rate and full diversity, 4 modulated symbols have to be emitted during 2 symbol times. Many Space-Time codes have been designed for 2×2 channels but we will focus especially on the Golden and the Silver codes which are the two best codes for this configuration.

1) The Golden code: The Golden Code [10] has the best performances on 2×2 MIMO Rayleigh fading channels. The encoded symbols are sent on two polarizations ($pola_1$,$pola_2$) during two symbol times (T_1 ,T_2). The codeword matrix of the Golden code is:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{pola_1,T_1} & \mathbf{X}_{pola_1,T_2} \\ \mathbf{X}_{pola_2,T_1} & \mathbf{X}_{pola_2,T_2} \end{bmatrix}$$

$$\mathbf{X} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\left(S_1+\theta S_2\right) & \alpha\left(S_3+\theta S_4\right) \\ i\overline{\alpha}\left(S_3+\overline{\theta} S_4\right) & \overline{\alpha}\left(S_1+\overline{\theta} S_2\right) \end{bmatrix} \quad (10)$$

where $\theta = \frac{1+\sqrt{5}}{2}$, $\overline{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1+i+i\theta$, $\overline{\alpha} = 1+i+i\overline{\theta}$ and S_1, S_2, S_3, S_4 are four modulated symbols. The codeword matrix of the Golden code has a full rank of 2 which ensures the maximum diversity. It achieves a full rate of 2 symbols/cu because 4 symbols are transmitted during 2 symbol times. Moreover, the determinant (corresponding to the coding gain on Rayleigh fading channel) is proportional to $\frac{1}{5}$ which is the highest value obtained for a 2×2 Space-Time code.

2) The Silver code: Silver code [11] has performances very close to the Golden code but has also the advantage of having a reduced decoding complexity due to its particular structure. The codeword matrix of the Silver code is:

$$\mathbf{X} = \begin{bmatrix} S_1+Z_3 & -S_2^*-Z_4^* \\ S_2-Z_4 & S_1^*-Z_3^* \end{bmatrix}$$

$$\begin{bmatrix} Z_3 \\ Z_4 \end{bmatrix} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1+i & -1+2i \\ 1+2i & 1-i \end{bmatrix} \begin{bmatrix} S_3 \\ S_4 \end{bmatrix} \quad (11)$$

where S_1, S_2, S_3, S_4 are four modulated symbols. The determinant of this code is proportional to $\frac{1}{7}$.

C. Design criterion

The optical channel model is not a Rayleigh channel as in wireless transmission. Therefore, the performances of the Space-Time codes might not be based on the same criteria. In order to understand the performances of the different Space-Time codes, it is important to point out a criterion on which their performances are based. We use the upperbound of the paire wise error probability defined in [13]:

$$P_e \leq \sum_{X_1\in\mathcal{C}} \sum_{\substack{\mathbf{X}_2\in\mathcal{C} \\ \mathbf{X}_1\neq\mathbf{X}_2}} \exp\left(-\frac{E_H\left[\|H\left(X_1-X_2\right)\|^2\right]}{8\sigma^2}\right) \quad (12)$$

where $\|\mathbf{X}\| = \sqrt{trace\left(\mathbf{X}\mathbf{X}^{\dagger}\right)}$ and σ^2 is the noise standard deviation. The norm $\|\mathbf{H}\left(\mathbf{X}_1 - \mathbf{X}_2\right)\|$ represents the Euclidian distance between two codewords when modified by the transfer matrix $\mathbf{H}$.

The minimum distance d_{min} of a Space-Time code is the minimum Euclidian distance between two different codewords. However the matrix $\mathbf{H}$ modifies the codeword constellation and reduces the minimum distance between the codewords which increase the error probability. The minimum distance becomes:

$$d_{min} = \min_{\mathbf{X}_1, \mathbf{X}_2} \left(\|\mathbf{H}\left(\mathbf{X}_1 - \mathbf{X}_2\right)\|\right) \tag{13}$$

The Space-Time code having the best performances under the optical channel is the one having the highest minimum distance d_{min}. For every PDL values, the transfer matrix $\mathbf{H}$ of Eq.3 is function of the random angles α and β thus the minimum distance of the code is only function of β because R_α is a rotation matrix which does not change d_{min}. Fig.3 shows the minimum distance of the Silver and the Golden codes for different angle β. The the Sezginer-Sari [12] code is also introduced and compared with the two previous codes. Its construction is very similar to the Silver code but its performances are smaller on the wireless channel.

Fig. 3. Minimum distance of Space-Time codes depending on the rotation angle β of the transfer matrix for a QPSK constellation

d_{min} shows a $\frac{\pi}{2}$ periodicity. As the rotation matrix angle is uniformly distributed over $[0, 2\pi]$, we can deduce the average minimum distance $E_{\mathbf{H}}\left[d_{min}\right]$ of each code on the optical channel by averaging d_{min} on $\left[0, \frac{\pi}{2}\right]$ (see Fig.4).

We can notice that the Silver code has the highest minimum distance and thus, is outperforming the Golden code and the Sezginer-Sari code in this configuration. The criterion of the determinant valid for a Rayleigh channel is no more standing here. Indeed although Silver code has a smaller minimum determinant than the Golden code, it is having a higher minimum distance. With the considered PDL values, Silver code minimum distance is slightly degraded. For higher values of PDL, the minimum distance of the Silver code is decreasing

Fig. 4. Average minimum distance of Space-Time codes function of PDL for a QPSK constellation

too, however such PDL values are too high to be considered in real transmissions.

V. Simulation and Results

We consider an OFDM system leading to the channel model of Eq.5. The transfer matrix is modeled by Eq.3 and the modulated symbols belong to a QPSK constellation. In optical transmission systems, the FEC requirement is to provide an output $\mathrm{BER} < 10^{-12}$ for an input $\mathrm{BER} > 10^{-3}$ therefore, the SNR penalties at $\mathrm{BER} = 10^{-3}$ are computed for different PDL values on Fig.5.

Fig. 5. SNR penalties introduced by PDL at $\mathrm{BER} = 10^{-3}$ with and without Space-Time coding for deterministic PDL

We can clearly see the important penalty reduction brought by Space-Time coding. When there is no PDL, those Polarization-Time codes do not introduce any penalties as they are by construction redundancy free. Therefore, the use of a Polarization-Time code is always profitable. The simulation result are in agreement with the previous section analysis, indeed the Silver code performs better than the Golden code. In this model, although the angles between the PDL elements and

Fig. 6. Performances of the Golden and the Silver code when PDL is considered Gaussian

Fig. 7. SNR penalties introduced by the Gaussian PDL at BER$= 10^{-3}$ with and without Space-Time coding

the polarization states are chosen random, the PDL values have been considered deterministic. It means that the two equivalent channels fading λ_1 and λ_2 are always equal. However it has been shown that PDL is a stochastic process following a particular distribution. In [7], the PDL has been measured and follows a Gaussian distribution. Therefore, we now consider a PDL model in which Γ is the standard deviation of a zero mean Gaussian distribution. We measure on Fig.6 the BER of the transmission with a PDL $=$ 3dB with and without Polarization-Time coding. We notice that when the PDL is no more deterministic, the diversity is lost. Using a PT code will have the double advantage of bringing a coding gain and recovering the full diversity. The Silver code have almost the same performances as the case without PDL, moreover it keeps outperforming the Golden code in the case of fluctuating values of PDL (see Fig.7).

VI. Conclusion

We have introduced the Golden code and the Silver code as Polarization-Time codes in order to mitigate the PDL impairments in coherent optical 2×2 polarization multiplexed OFDM systems. Those codes both offer no redundancy, with a full rate of 2 symbol/cu. Polarization-Time coding is useless against PMD but can very efficiently mitigate the PDL impairments. Indeed, important coding gains have been observed considering the PDL determinist or stochastic. From the optical channel model we have shown that performances are no more based on the determinant criterion as in Rayleigh fading channel and that the Silver code outperforms the Golden code in this configuration.

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