Efficient Interleaving of FEC Codewords for Optical PSK Systems

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Abstract *Differential encoding is required for phase modulation optical transmission systems but leads to higher bit error rate. We propose a construction of the codeword of the forward error correction allowing performance enhancement and complexity decrease.*

Introduction

Phase shift keying (PSK) modulation formats are very promising for high bit-rate optical transmissions[1,2]. Forward error correction (FEC) techniques[3] have been introduced to increase system margins. Differential encoding is necessary to deals with the lack of absolute phase reference in direct-detection DPSK systems and the inability of phase recovery algorithms to correct phase noise accumulation in coherent detection systems[4]. However differential encoding leads to higher bit error rates (BER) because each transmission error corrupts multiple consecutive bits. As a consequence, the FEC performances are affected.

We present a novel construction of the codeword from the FEC allowing, by a new decoding method, a considerable reduction of the decoding complexity and offering a significant coding gain. The work has been realized in the case of a QPSK modulation format but can be applied on any system using differential encoding.

Differential encoding

In differential encoding, the information is encoded in the transition between the states of the constellation. With a QPSK modulation the information is carried by the phase shift between two successive symbols. Moreover, a Gray mapping is generally applied in order to reduce BER. A major issue of differential encoding is that a single transmission error corrupts two transitions (see Fig.1); therefore the two information symbols encoding those transitions are erroneous. The number of bits encoding the symbol depends on the number of bits required to encode a transition in the constellation (i.e two bits form a symbol in a QPSK modulation).

Fig. 1: Principle of transmission error on QPSK systems using differential encoding (α_1, α_2, α_3, α_4 are the four states of the QPSK constellation)

The error can be expressed in number of quadrant Nerr. For example, an error changing a transition "11" (2 quadrants) to "01" (1 quadrant) is an error of -1 quadrant. Note that the sum of the quadrant error due to consecutive transmission errors is null. For instance in Fig1, a -1 quadrant error is followed by a +1 quadrant error. The k+1 quadrant errors produced by k transmission errors follow the expression:

$$\sum_{i=1}^{k+1} \mathrm{Nerr_i} \quad \mathrm{mod}(4) = 0 \qquad (1)$$

where $\mathrm{Nerr_i}$ is the ith of the k+1 errors.

Symbol interleaving

The two consecutive erroneous information symbols produced by a single transmission error are usually located on a single FEC codeword. We propose here, a symbol interleaving of different codewords such that consecutive information symbols belong to different codewords (see Fig.2).
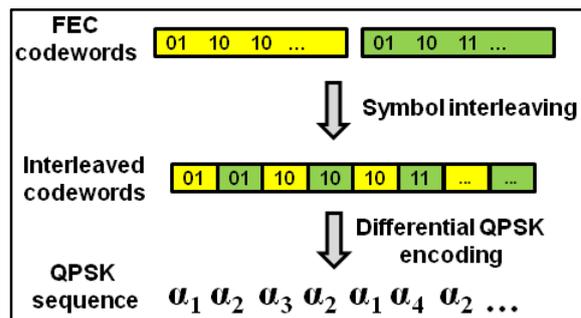
Fig. 2: Symbol interleaving of two codewords in a QPSK transmission

Fig. 3: Comparison of FECs with and without symbol interleaving

The interleaving can be applied on more than two codewords but only the two codewords interleaving is presented in this paper. Considering this construction, a transmission error produces only one erroneous

symbol on each two codewords instead of two erroneous symbols on a unique codeword. Hence the required error correction capability of the FEC can be reduced. Fig. 3 presents the compared performances of FECs with and without symbol interleaving for different FEC schemes. A FEC with symbol interleaving has the same performances than the code twice longer correcting twice more errors. The interleaving only reduces the differential encoding penalties and the obtained coding gain depends on the FEC type. Note that, as non-binary FECs like Reed Solomon (RS) codes are by nature slightly sensitive to differential encoding penalties, the advantages of the symbol interleaving are not significant.

Decoding complexity reduction

The symbol interleaving can also be used to reduce the decoding complexity. From the previous construction, we decode only one of the two codewords with the classical FEC decoder and entirely deduce the decoding of the other one. From the FEC decoding, we obtain the position and the value of the error on the first codeword. The error value of the second codeword can be directly deduced using Eq. (1). Moreover, as symbol errors occur per pair, thanks to our construction, the FEC decoding will always correct one of the two consecutive erroneous symbols. The second erroneous symbol is one of its neighbors and belongs to the other codeword. The only uncertainty is which the erroneous neighbor is (see Fig. 4).
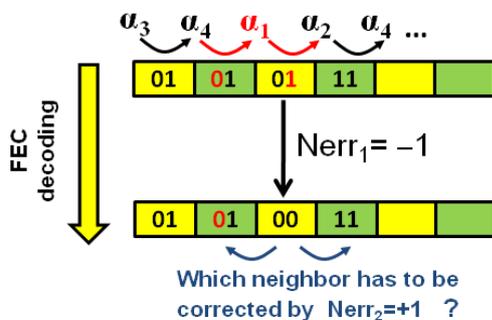
Fig. 4: After the FEC decoding of the first codeword, the value of the error on the second codeword can be deduced. Its position can either be the right neighbor or the left neighbor of the error detected on the first codeword.

We decide to correct the neighbor corresponding to the least reliable received symbol. The reliability is estimated computing the Log Likelihood Ratio (LLR)[5]. We check if the correction gives a valid codeword computing the syndrome[5] of the corrected codeword. If the correction is not valid (i.e the computed syndrome is not null), a classical FEC decoding is processed for the second codeword. Note that, no valid codeword can be found when the wrong neighbor has been chosen or when many consecutive transmission errors have occurred. If the syndrome

computation complexity is considered negligible, the complexity of our method only depends on the number of FEC decoding performed (i.e. the number of times that our correction didn't produce a valid codeword). The complexity reduction is estimated measuring the average number of FEC decoding realized in order to decode the two codewords.

Fig. 5: Percentage of complexity reduction depending on the post-FEC bit error rate

Fig. 5 plots the total complexity reduction on the output BER. An error free transmission (BER<10^{-12}) can be realized decoding only one codeword which is equivalent to a 50% decoding complexity reduction. The BCH codes can reach a 50% complexity reduction because the needed input BER to achieve an error free transmission is high. Therefore there are few consecutive errors and few chances not to recognize the erroneous QPSK symbols, which imply few FEC decoding of the second codeword. The product codes are more powerful (i.e higher coding gain) and work at lower SNR as shown on Fig. 3. So the FEC decoding of the second codeword happens more often, a 45% decoding complexity is only reachable. This decoding method is as efficient with non binary FECs, like RS codes.

Conclusions

Symbol interleaving has been introduced in order to mitigate the differential encoding penalties. A decoding algorithm based on this construction has also been presented offering an important complexity reduction going until 50%. Our scheme can be very profitable and easily implemented in high bit-rate optical transmission systems.

References
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