# Code Construction for the Selective TDMA Cooperative Broadcast Channel

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Abstract—In this paper, the selective time division multiple access (S-TDMA) strategy is studied in the downlink channel. This strategy consists in transmitting data to the user with the largest capacity. The diversity and multiplexing gains that can be achieved by this sub-optimal strategy are evaluated and then compared to the optimal gains over the broadcast channel. Codes construction is then proposed to achieve the diversity multiplexing tradeoff (DMT) of the S-TDMA. These codes are extended to the scenario of cooperating asynchronous broadcasting base stations where new codes that are suitable for this scenario are proposed and analyzed.

#### I. INTRODUCTION AND MOTIVATIONS

In this paper, we consider the multiuser downlink context with one or more cooperating base stations (BS). We assume that each base station has M transmit antennas and the K users has N antennas each.

In the single cell context, when perfect channel state information (CSI) is assumed at both the BS and receivers, it is well known that the Dirty Paper Coding technique (DPC) achieves the maximum sum capacity. This technique is the most efficient strategy that allows a base station to transmit data to multiple users at same time. In this case, the multiuser BC channel is equivalent to a  $M \times KN$  MIMO system in term of its sum capacity [1]. But, the implementation of DPC brings high complexity to both the transmitter and the receiver. In addition, full CSI is required at the transmitter side which is not practical in a real system.

On the other hand, when no CSI is available at the transmitter and the channels of all receivers are statistically identical, then, the BC channel is degraded in any order and TDMA is the optimal strategy. In this case, the multiuser BC channel is equivalent to a single user  $M \times N$  MIMO channel. As we can see, there is a huge gap between the multiuser gains with and without transmit CSI. Since lack of CSI does not lead to multiuser gains and since perfect CSIT is not feasible, it is interesting to assume the knowledge of partial CSI at the transmitter. A simple technique consists in transmitting to the user with the strongest capacity [2]. In the following, this strategy will be referred as *selective TDMA*. It has been shown in [3] that for the BC with single antenna users, when no directional information is available at the transmitter, the selective TDMA is the optimal strategy that maximizes the sum capacity. In [4], the Diversity Multiplexing Tradeoff (DMT) for scalar and vector BC were derived and optimal schemes that achieve these DMT were proposed.

When multi-cell scenario is considered, it has been shown recently in [5], that it is better to perform cooperation between base stations rather than the traditional handover in order to increase power and rate efficiency. Then, the base stations cooperate to send data to a selected user on the cell-boundary. In this context, the different base stations are not necessarily co-located. This implies a lack of synchronisation between the different base stations. That's why, it is of interest to construct delay tolerant space-time code suitable for this scenario.

The remainder of this paper is organized as follows. In section II, we define the system model and and the notations used in this paper. In section III, we compare the selective TDMA strategy to the DPC approach in term of diversity and multiplexing gain. Space time coding that achieve the DMT for selective TDMA in the single cell-context are proposed in section IV. New code construction suitable for the multicell scenario with cooperating base stations and single antenna users is proposed in section V. Finally, section VII concludes this paper.

#### **II. SYSTEM MODEL**

We consider a K receiver multiple-antenna broadcast channel in which the transmitter has M antennas and each receiver has N antennas (fig 1). The received signal  $Y_k$  for user k is given by

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k \quad k = 1, \dots, K$$

where  $\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_K$  are the channel matrices (with  $\mathbf{H}_i \in \mathbb{C}^{N \times M}$ ) of user 1 to K, with i.i.d unit variance Gaussian entries. The vector  $\mathbf{x} \in \mathbb{C}^{M \times 1}$  is the transmitted signal, and  $\mathbf{n}_1, \ldots, \mathbf{n}_k$  are independent complex Gaussian noise terms with unit variance. The input must satisfy a transmit power constraint of P, *i.e.*  $\mathbb{E}[||x||^2] \leq P$ . We assume that each receiver has perfect knowledge of its own channel matrix.

In terms of notation, we use  $H^{\dagger}$  to indicate the conjugate transpose of matrix **H** and  $||\mathbf{H}||$  to denote the matrix norm of

$$\|\mathbf{H}\| = \sqrt{\lambda_{\max}(\mathbf{H}^{\dagger}\mathbf{H})}$$



Fig. 1. A broadcast channel: K users with N antennas each and a transmitter with M antennas

## III. ASYMPTOTIC GAINS: SELECTIVE TDMA VERSUS DPC

In this section, we compare in the single cell scenario the selective TDMA strategy to the DPC in term of diversity and multiplexing gains. First, we define the outage event. The optimal DMT for the selective TDMA strategy is computed, and the maximal diversity and multiplexing gains are then compared to the optimal diversity and multiplexing gains that can be achieved in the BC case.

#### A. Outage Definition

In MIMO broadcast channel, when full CSI is assumed at the transmitter, without any assumption on the transmission rates, the outage analysis is not meaningful. In this case, the rate can be adapted to the link in order to prevent outage event to occur. However, if we impose conditions on the transmitted rate, the outage probability can be defined with respect to a fixed rate constraint.

For DPC schemes, the outage occurs when the rates vector falls out the capacity region of the BC. This means that the required sum rate R exceeds the optimal sum rate capacity of the BC. For the selective TDMA case, outage occurs when the user with the strongest capacity does not support the required rate.

#### B. Optimal Tradeoff for the selective TDMA

*Proposition 1:* The Diversity Multiplexing Tradeoff (DMT) of a broadcast channel when the selective TDMA strategy is used is given by

$$d_{\text{TDMA}}(r) = K \, d_{M,N}(r) \tag{1}$$

where r is the total multiplexing gain;  $d_{M,N}(r)$  is the diversity multiplexing tradeoff of  $M \times N$  MIMO Rayleigh channel.

*Proof:* The DMT has been defined in [6]. For a MIMO  $M \times N$  channel, it is given by the piecewise linear function

$$d_{M,N}(r) = (M-r)(N-r)$$

As defined in [2], the TDMA strategy that we consider consists in transmitting only to the user with the maximum capacity such as

$$i^* = \max_{i=1...K} I(X, Y | \mathbf{H}_i)$$

where  $i^*$  corresponds to the selected user and  $I(X, Y | \mathbf{H}_i)$  denotes the single user mutual information. The outage probability is then

$$P_{\text{out}}(R) = \operatorname{Prob}\left\{\max_{i=1...K} I(X, Y|\mathbf{H}_i) < R\right\}$$
$$= \prod_{i=1}^{K} \operatorname{Prob}\left\{I(X, Y|\mathbf{H}_i) < R\right\}$$
$$= \operatorname{SNR}^{-K d_{M,N}(r)}$$

which completes the proof.

#### C. Asymptotic DPC diversity and multiplexing gains

*Proposition 2:* For the broadcast channel, the optimal diversity gain that can be achieved *under a fixed sum rate constraint* is

$$d_{\rm DPC} = MKN \tag{2}$$

and the optimal multiplexing gain is  $\min(M, KN)$ .

*Proof:* The optimal sum capacity of BC is bounded by

$$C_{\text{TDMA}} \le C_{\text{BC}}(H; P) \le M \log(1 + \frac{P}{M} \left\| H \right\|_{\max}^2) \quad (3)$$

The first inequality follows from the fact that the selective TDMA is a sub-optimal strategy. The second inequality is a result of [theorem 1] in [2], where

$$|H||_{\max}^2 = \max_{k=1...K} ||H_k||^2.$$

The outage probability is therefore bounded by

$$P_{\text{bound}} \le P_{\text{out}} \le P_{\text{TDMA}}$$
 (4)

where  $P_{\text{TDMA}} \doteq \text{SNR}^{-MKN}$  from proposition 1 and

$$P_{\text{bound}} = \operatorname{Prob}\left\{ M \log(1 + \frac{P}{M} \|H\|_{\max}^2) \le R \right\}$$
  
$$\doteq \operatorname{SNR}^{-MKN}$$
(5)

We prove the result in eq (5) using the pdf of the eigen exponent associated to  $\|\mathbf{H}\|_{\max}^2$  given in lemma 1.

Lemma 1: Let  $\tilde{\alpha}$  be the eigen exponent associated to  $\|\mathbf{H}\|_{\max}^2$ , then

$$p(\tilde{\alpha}) \doteq \text{SNR}^{-MKN \ \tilde{\alpha}} \tag{6}$$

The proof of this lemma is detailed in appendix A.

This implies that the maximal diversity order that can be achieved using DPC is MKN. The optimal multiplexing gain has been already computed by Jindal in [2] which completes the proof.

#### D. Asymptotic gains: DPC versus TDMA

The comparison of DPC versus selective TDMA is summarized in table I. As we can see, both strategies can achieve the same diversity order which is MKN. But the throughput degradation that results from using sub-optimal method rather than optimal DPC strategies impacts the maximal multiplexing gain. The optimal multiplexing gain with DPC techniques is  $\min(M, KN)$ . However, the maximal multiplexing gain that can be achieved with the selective TDMA is  $\min(M, N)$ . Regarding this comparison, we can see that for a cell with a large number of users, the selective TDMA seems to be an optimal strategy in terms of asymptotic gains.

Strategy	Diversity	Mux gain
DPC	MKN	$\min(M, KN)$
TDMA	MKN	$\min(M,N)$

TABLE I

COMPARISON OF DPC AND TDMA IN TERMS OF ASYMPTOTIC GAINS

#### IV. SELECTIVE TDMA IN SYNCHRONOUS SYSTEM: ACHIEVING DMT USING PERFECT CODES

Let  $i^*$  be the selected user. Then, the equivalent system model corresponds to

$$\mathbf{Y} = \mathbf{H}_{i^*} \mathbf{X} + \mathbf{n} \tag{7}$$

where  $\mathbf{Y}$  is the received signal at user  $i^*$ ,  $\mathbf{X}$  is the transmitted space time code,  $\mathbf{n}$  is the additive noise.

#### A. Single antenna users case

It has been shown in [3] that for the BC with single antenna users, when no directional information is available at the transmitter, the optimal strategy consists to select the user with the strongest capacity. In [4], it has been shown the one-layered perfect code achieves the DMT which is MK(1-r).

#### B. General case

The equivalent BC model is given in eq. (7). The system model is equivalent to a  $M \times N$  MIMO channel but with a different distribution than the classical Rayleigh MIMO channel. It is well-known that the perfect codes [7] are universal space time codes by construction with non-vanishing determinant (NVD). This implies that these codes achieve the DMT regardless of the channel distribution [8].

### V. NETWORK MIMO: DELAY TOLERANT SPACE TIME CODING

In this section, we consider two or more base stations, each with M transmit antennas, that cooperate to send data to a selected user on the cell boundary. Cooperation between base stations is performed instead of traditional handover in order to increase power and rate efficiency in the downlink broadcast channel [5]. Let T be the number of base stations, each with M transmit antennas, cooperating to transmit to the selected user with N receive antennas such that Q = TM is the total number of transmit antennas. With more than one

user on the cell boundary, the base stations can select the user with the largest capacity. Since the base stations are in different geometrical locations, the receiver experience different delay propagations. We suppose that the base stations are asynchronous by an integer multiple of the symbol period (by assuming that the fractional delays are absorbed in multipath, cf. [9] and references therein).

Because of the lack of synchronization, the design criteria of the space-time code formed by the different base stations should be modified accordingly [9]. In order for the new code to achieve full diversity with different delays, the difference between any two distinct code words should now remain full rank as one shifts the different rows arbitrarily. For example, consider the one-layer code for the 2 cooperating singleantennas BTS defined in section IV-A, given by

$$\left(\begin{array}{cc} x & 0 \\ 0 & \sigma(x) \end{array}\right),$$

with  $x = \alpha(s_1 + \theta s_2)$  and  $\sigma(x) = \overline{\alpha}(s_1 + \overline{\theta}s_2)$  for  $s_1$  and  $s_2$  from a QAM constellation  $\theta = \frac{1+\sqrt{5}}{2}$ ,  $\overline{\theta} = \frac{1-\sqrt{5}}{2}$ ,  $\alpha = 1 + i - i\theta$ ,  $\overline{\alpha} = 1 + i - i\overline{\theta}$  which was originally designed in [4] to achieve the optimal diversity-multiplexing tradeoff when used by a base station with M = 2 to send data to the strongest user with N = 1 receive antenna. One can show that this code will not achieve full diversity if used by two base stations each with M = 1 antenna and if there is a delay of one symbol between them

$$\left( egin{array}{ccc} 0 & x & 0 \\ 0 & \sigma(x) & 0 \end{array} 
ight).$$

To remedy this problem, the code can be modified as follows

$$rac{1}{\sqrt{2}}\left(egin{array}{cc} x & i\sigma(x) \ x & \sigma(x) \end{array}
ight),$$

where it is easily checked that it is delay tolerant (i.e., the difference between distinct code word matrices remains full rank when rows are shifted arbitrarily corresponding to arbitrary delays among the cooperating base stations).

To design a delay-tolerant space-time code of rate N symbol per channel use one can choose one of the optimized codes in [9]. However, the temporal length of the codes in [9] increases exponentially with the number of transmit antennas. The codes proposed in [10] are square (i.e., with temporal length equal to the number of transmit antennas) but their rate equals Qsymbol per channel use, which is not useful in our case since often Q = TM is often larger than N in practical scenarios. Therefore, we propose a small modification on the construction in [10] in order to fit our purpose. Recall that the main idea in the construction in [10] is to have the space-time code word matrix with each element different from any other matrix (such that the difference matrix has all its entries non zero) and then to multiply the upper triangular elements with different power of an algebraic or transcendental number  $\phi$  in order to make it full rank with arbitrary row shifts. Here we use rate N perfect space-time codes and layer them using repetition codes. For example, in the synchronous case, a one-layered perfect code achieves the optimal DMT [4]

$$\left(\begin{array}{cccc} x & 0 & \dots & 0 \\ 0 & \sigma(x) & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \sigma^{Q-1}(x) \end{array}\right)$$

One way to make the above code delay tolerant is the following

$$\frac{1}{\sqrt{Q}} \begin{pmatrix} x & \sigma(x) & \sigma^2(x) & \dots & \sigma^{Q-1}(x) \\ x & \sigma(x) & \sigma^2(x) & \dots & \sigma^{Q-1}(x) \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x & \sigma(x) & \sigma^2(x) & \dots & \sigma^{Q-1}(x) \end{pmatrix} \odot \mathbf{B}_Q$$

where  $\odot$  denotes the component-wise product and matrix  ${\bf B}$  is given by

$$\mathbf{B}_{Q} \triangleq \begin{pmatrix} 1 & \phi^{b_{1,2}} & \phi^{b_{1,3}} & \cdots & \phi^{b_{1,Q}} \\ 1 & 1 & \phi^{b_{2,3}} & \cdots & \phi^{b_{2,Q}} \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & \phi^{b_{Q-1,Q}} \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}$$

and the number  $\phi$  and its power  $b_{i,j}$  are chosen in accordance with rules in [10].

#### VI. NUMERICAL RESULTS

For illustration, we consider the case of a single-cell where a broadcast channel has 2 antennas (M = 2) and 2 users with 2 antennas at each receiver. As shown in proposition 1, the optimal DMT that can be achieved by using the S-TDMA strategy is 2(2-r)(2-r). This DMT can be achieved using a universal space-time code which is the Golden code [11] for this  $2 \times 2$  configuration. The transmitted signal to the best user is given by

$$\mathbf{X} = \begin{bmatrix} \alpha(s_1 + \theta s_2) & \alpha(s_3 + \theta s_4) \\ \gamma \bar{\alpha}(s_3 + \bar{\theta} s_4) & \bar{\alpha}(s_1 + \bar{\theta} s_2) \end{bmatrix}$$
(8)

with  $s_1$ ,  $s_2$ ,  $s_3$  and  $s_4$  denote the information QAM symbols.

In fig. 2, we compare for this antenna configuration, the S-TDMA strategy versus the DPC in terms of capacity. As expected from the theoretical results in table I, we can see that for this antenna configuration the maximal multiplexing gain that can be achieved using both strategies (DPC and selective TDMA) is the same and is equal to 2. But, there is a throughput degradation that results from the use of a sub-optimal strategy rather than optimal DPC. However, this throughput degradation does not affect the diversity order which is equal to 8 in fig. 3.

The performance of the S-TDMA strategy is also studied in term of frame error rate. By selecting the user with the strongest capacity and using the Golden code in fig.3, the optimal diversity order of 8 can be achieved. This explains the large gain obtained over time sharing case, where the diversity order is only 4.



Fig. 2. Comparison of the sum capacity of the DPC technique and the selective TDMA strategy for a BC with dual transmit antennas and 2 users with 2 Rx antennas



Fig. 3. Comparison of the frame error probabilities of the S-TDMA with a Golden code and the conventional TDMA for 2 bits spectral efficiency (QPSK)

The multi-cell case with 2 base stations is then considered. Each base station has a single antenna and both base stations cooperate in order to transmit data to a selected single antenna user at the cell boundary. In figure 4, we show that when the BS are not synchronous, the one-layer code is no more suitable when the delay is taken into consideration. This is traduced by a loss in the diversity gain and the diversity order in this case is reduced to 2. When the delay tolerant space time code proposed in section V is used for this configuration, the optimal diversity gain of 4 remains the same as in the synchronous case. This explains the large gap in SNR of 10 dB at a PER of  $10^{-2}$ .

#### VII. CONCLUSION

In this paper, we consider the selective TDMA strategy which consists to transmit data to the user with the strongest capacity. We derive the DMT of the Broadcast Channel (BC) when this strategy is used. The asymptotic gains in term



Fig. 4. Comparison of the frame error probabilities of the new-delay tolerant space time code and the one-layer perfect code with an asynchronisation delay of one time symbol

of diversity and multiplexing gain are then compared to the optimal asymptotic gains that can be achieved over a broadcast channel (BC). We show that both strategies achieve the maximal diversity gain. But, the throughput degradation that results from using sub-optimal method such as selective TDMA rather than optimal DPC strategies impacts the maximal multiplexing gain. Regarding this comparison, we conclude that for BC with a large number of users, the selective TDMA seems to be an optimal strategy in terms of asymptotic gains. We show also that the perfect codes achieve the DMT of the S-TDMA strategy.

We extend this study to the multi-cell scenario, where the base stations cooperate to send data to a user on the cell boundary. In this case, the base stations are not co-located and not necessarily synchronized. To resolve the problem of lack of synchronisation, new delay tolerant space time codes are proposed.

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#### APPENDIX A

#### **PROOF OF LEMMA** 1

Let  $\alpha_1, \ldots, \alpha_q$  denotes the  $q = \min(M, N)$  eigen exponents of  $H_k$   $(k = 1 \ldots K)$  and  $\beta_k$  denotes the minimal eigen exponent.

Following [6], the joint pdf is given by

$$p(\alpha_1,\ldots,\alpha_q) \doteq \mathrm{SNR}^{-\sum_{i=1}^q (2i-1+|M-N|)\alpha_i}$$

with  $\alpha_1 \geq \ldots \geq \alpha_q = \beta_k$ . Then,

$$p(\beta_k) \doteq \int_{\mathcal{D}} \mathrm{SNR}^{-\sum_{i=1}^{q} (2i-1+|M-N|)\alpha_i} d\alpha_1 \dots d\alpha_{q-1}$$
  
$$\doteq \mathrm{SNR}^{-(|M-N|-1)\beta_i} \mathrm{SNR}^{-\min_{\alpha \in \mathcal{D}} f(\alpha_1, \dots, \alpha_{q-1})}$$
  
with  $\mathcal{D} = \{\alpha_1 \ge \dots \ge \alpha_q = \beta\}.$ 

By using Laplace method, the optimal solution is achieved for  $\alpha_1 = \ldots = \alpha_{q-1} = \beta_k$ 

$$p(\beta_k) \doteq \mathrm{SNR}^{-MN \ \beta_k}$$

As 
$$\tilde{\alpha} = \min_{k=1...K} \beta_k$$
, then  

$$p(\tilde{\alpha}) = \int_{\mathcal{A}} p(\tilde{\beta}_1, \dots, \tilde{\beta}_K) d\tilde{\beta}_1 \dots \tilde{\beta}_K$$

with  $(\tilde{\beta}_1, \ldots, \tilde{\beta}_K)$  is the ordered combination of  $(\beta_1, \ldots, \beta_K)$ and  $\mathcal{A} = \{\tilde{\beta}_1 \geq \ldots \geq \tilde{\beta}_K = \tilde{\alpha}\}$ . Since the  $\beta_i$  are independent random variables and the order transformation is a permutation with a jacobian |J| = 1, this implies that

K

$$p(\tilde{\beta}_1, \dots, \tilde{\beta}_K) \doteq K! \prod_{k=1}^K \text{SNR}^{-MN \ \tilde{\beta}_k}$$

Finally, we obtain

Then,

$$p(\tilde{\alpha}) \doteq K! \text{ SNR}^{-MKN \tilde{\alpha}}$$

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