

# On the performance of the Golden code in BICM-MIMO and in IEEE 802.11n cases

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**Abstract**—In  $2 \times 2$  MIMO systems, the Golden code (GC) [1] was proposed when no binary outer code is applied at the transmitter. This code is optimal since it achieves full rate, full diversity and the Diversity Multiplexing Tradeoff [2], and it preserves the mutual information. We propose in this paper to study the performance of this code compared to Spatial Division Multiplexing (SDM) [3] in a BICM-MIMO system. We derive the Pairwise Error Probability (PER) in this context and we assess our results over a Rayleigh channel and in the 802.11n case. We show that the GC is also optimal for  $2 \times 2$  BICM-MIMO systems. However, the impact of the additional gain provided by the Golden code is reduced when using binary outer code with high free distance case, as for instance in IEEE 802.11n standard.

**Index Terms**—Bit Interleaved Coded Modulation (BICM), Orthogonal Frequency Division Multiplexing (OFDM), Spatial Division Multiplexing (SDM), Space Time Block Coding (STBC), Golden code (GC), diversity.

## I. INTRODUCTION

The objective of IEEE802.11n standardization is to achieve 100Mbps in top of MAC layer while still being backward compatible with IEEE802.11a/g, which results in a maximum PHY rate of 130Mbps. This significant rate increase compared to other IEEE802.11 standards such as IEEE802.11a, whose maximum PHY rate is 54Mbps, is enabled by the introduction of multiple antennas at the Access Point and at the Mobile Terminal. These multiple antennas are used to increase the peak data rate, but also to derive benefit from spatial diversity in order to ensure for instance a larger range of operation for full home coverage, or to better address outdoor hotspot environments. Another feature of IEEE802.11n consists in addressing handsets specificities, such as a small number of antennas. To accommodate various antenna configurations, the definition of PHY modes is based on the transmission of a number of spatial streams, varying from one to four, that is limited by the minimum number of transmit and receive antennas. For range increase, Space-Time Block Coding (STBC) or/and Cyclic Delay Diversity (CDD) can be applied to map the spatial streams on different transmit chains. Most of pre 802.11n products use two transmit and two or three receive

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antennas, which limits the number of spatial streams to two. In this configuration, only full diversity or full rate modes have been defined, which motivates the study of full rate full diversity space-time codes.

The Golden code was proposed in [1] for  $2 \times 2$  MIMO configuration, in order to fulfill the design criteria proposed by Tarokh in [4] and in order to achieve the Diversity Multiplexing Tradeoff (DMT) [2]. This code is the optimal  $2 \times 2$  space time code since it achieves full rate and full diversity, preserves the mutual information and achieves the DMT: by construction, the Golden code has a non vanishing determinant [1].

In this paper, we propose to study the performance of the Golden code compared to SDM (uncoded MIMO scheme) in a BICM-MIMO system. In section II, we derive the pairwise error probability for a BICM-STBC system. We show that the design criteria for BICM-STBC are the same as in MIMO case [4], i.e. the rank and the determinant criteria. We deduce that the Golden code is also optimal for BICM-MIMO systems. However, the impact of the additional gain provided by the Golden code depends on the robustness of the outer binary code. We assess this result over Rayleigh channels in II-C. Then, we describe the IEEE 802.11n standard in III-A in order to assess the performance of the Golden code when it is directly applied to this standard in III-B.

## II. THE BICM-STBC SYSTEM

In this section, we focus on two space time codes: spatial division multiplexing (SDM) and the Golden code, for  $2 \times 2$  MIMO configuration. By extension of BICM [5] to MIMO systems, we derive the error probability of these space time codes when concatenated to an outer binary code.

### A. System Model

The block diagram of the system is depicted in Fig. 1. During transmission, the binary information elements  $\underline{b}$  are first encoded by a binary code of rate  $R_c$  e.g. a convolutional code, and then interleaved by a bit interleaver  $\pi$ . The coded and interleaved sequence  $\underline{c}$  is fed into the  $2^m$ -QAM gray mapper and is mapped onto the signal sequence  $\underline{x} \in \mathcal{X}$ . The resulting symbols are coded by an algebraic space time block code with spreading factor  $s$  and code generator matrix  $G$ .

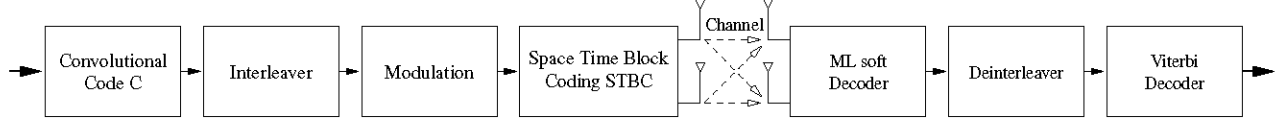


Fig. 1. BICM STBC system

In the following, the vectorized notations are used for simplicity. For the  $2 \times 2$  configuration, we focus on two space-time codes. The SDM case corresponds to  $s = 1$  and  $G = I_{n_t}$ . In the Golden code case,  $s = 2$  and the vectorized generator matrix is given by

$$G = \begin{pmatrix} \alpha & \alpha\theta & 0 & 0 \\ 0 & 0 & i\bar{\alpha} & i\bar{\alpha}\bar{\theta} \\ 0 & 0 & \alpha & \alpha\theta \\ \bar{\alpha} & \bar{\alpha}\bar{\theta} & 0 & 0 \end{pmatrix}$$

where  $\theta = \frac{1+\sqrt{5}}{2}$ ,  $\bar{\theta} = \frac{1-\sqrt{5}}{2}$ ,  $\alpha = 1+i-i\theta$  and  $\bar{\alpha} = 1+i-i\bar{\theta}$ . The coded codewords are finally transmitted on a multiple antenna channel  $\mathbf{H} = [h_{i,j}]$  with  $n_t$  transmit antennas and  $n_r$  receive antennas;  $h_{i,j}$  denotes the Rayleigh fading coefficients between transmit antenna  $j$  and receive antenna  $i$  with  $h_{i,j} \sim \mathcal{CN}(0, 1)$ .

The bit interleaver can be modeled as  $\pi: k' \rightarrow (k, i)$ , where  $k'$  denotes the original ordering of the coded bits  $c_{k'}$ ,  $k$  denotes the time ordering of the MIMO codewords  $X_k$  where  $X \in \mathcal{X}^{s n_t}$  and  $i$  indicates the position of the bits  $c_{k'}$  in the codeword.

At the receiver, the vectorized received signal is given by

$$Y = \mathbf{H}_e \mathbf{G} X + Z \quad (1)$$

where  $Z$  is the complex Gaussian noise  $Z \sim \mathcal{CN}(0, N_0 I_{n_r})$ , and  $\mathbf{H}_e$  denotes the equivalent block diagonal channel. For the SDM case,  $\mathbf{H}_e = \mathbf{H}$ , and for the Golden code

$$\mathbf{H}_e = \begin{bmatrix} \mathbf{H} & 0 \\ 0 & \mathbf{H} \end{bmatrix} \quad (2)$$

The ML soft decoder generates for each coded bit  $c_{k,i}$  two metrics:  $\lambda_{c_k=0}^i$  and  $\lambda_{c_k=1}^i$ . These metrics correspond to the log-MAP computed over one codeword, and are given by :

$$\lambda^i(c_k) = \log \sum_{X \in \mathcal{X}_{c_k}^i} p(Y | \mathbf{H}_e, X) = \log \sum_{X \in \mathcal{X}_{c_k}^i} \exp^{-\|Y - \mathbf{H}_e \mathbf{G} X\|^2}$$

$$\approx \min_{X \in \mathcal{X}_{c_k}^i} \|Y - \mathbf{H}_e \mathbf{G} X\|^2 \quad (3)$$

where  $\|\cdot\|^2 = (\cdot)^H (\cdot)$  is the Euclidean distance.  $\mathcal{X}_b^i$  denotes the constellation subset

$$\mathcal{X}_b^i = \{X \in \mathcal{X}^{s n_t} = \underbrace{\mathcal{X} \times \dots \times \mathcal{X}}_{s n_t} : l^i(X) = b\}$$

and  $l^i(X)$  is the  $i^{\text{th}}$  bit of the codeword  $X$ . For low complexity algorithms, these metrics can be computed using the list sphere decoder [6].

Then, the metrics associated to the interleaved bits are deinterleaved. Finally, the  $\lambda$  metrics are used by the Viterbi decoder

to decode the information bits by finding the shortest path in the trellis according to

$$\hat{c} = \arg \min_{c \in \mathcal{C}} \sum_{k'} \lambda(c_k^i) \quad (4)$$

### B. Error Probability Derivation

In [7], the pairwise error probability of BICM-STBC-OFDM using an orthogonal space time code such as the Alamouti code was derived. In this section, we use the same notations in order to derive the pairwise error probability for BICM-STBC using the SDM and the Golden code as space time block codes. Assuming that the code sequence  $\underline{c}$  is transmitted and  $\hat{c}$  is detected, the PEP given the channel knowledge can be written as

$$P(\underline{c} \rightarrow \hat{c} | \mathbf{H}) = P \left( \sum_{k'} \min_{x \in \mathcal{X}_{c_k}^i} \|Y(k) - \mathbf{H}_e \mathbf{G} X(k)\|^2 \leq \sum_{k'} \min_{x \in \mathcal{X}_{\hat{c}_k}^i} \|Y(k) - \mathbf{H}_e \mathbf{G} X(k)\|^2 \right) \quad (5)$$

Let  $d_{free}$  be the minimum Hamming distance of the convolutional code. Let us assume that the distance between the incorrect path associated to  $\hat{c}$  and the correct one associated to  $\underline{c}$  is  $d_{free}$ .  $\mathcal{X}_{c_k}^i$  and  $\mathcal{X}_{\hat{c}_k}^i$  are equal to one other for all  $k'$  except for  $d_{free}$  distinct values of  $k'$ . Then, only  $d_{free}$  terms are different in the inequality (5). Let us denote in the following  $\tilde{X}(k)$  and  $\hat{X}(k)$  as:

$$\tilde{X}(k) = \arg \min_{X \in \mathcal{X}_{c_k}^i} \|Y(k) - \mathbf{H}_e \mathbf{G} X(k)\|^2 \quad (6)$$

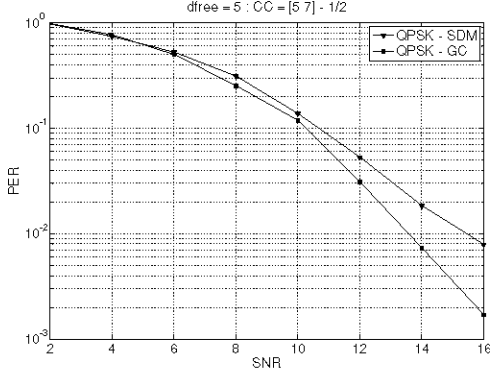
$$\hat{X}(k) = \arg \min_{X \in \mathcal{X}_{\hat{c}_k}^i} \|Y(k) - \mathbf{H}_e \mathbf{G} X(k)\|^2 \quad (7)$$

where  $\mathcal{X}_{c_k}^i$  is the complementary set of  $\mathcal{X}_{\hat{c}_k}^i$ . The PEP can be written as

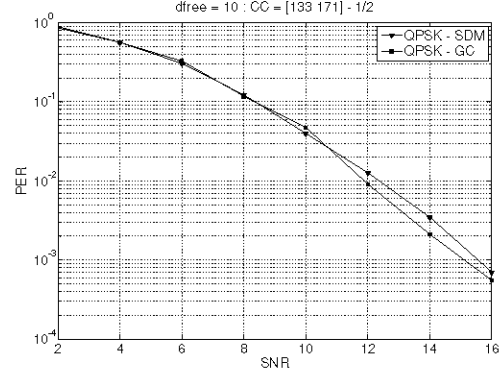
$$P(\underline{c} \rightarrow \hat{c} | \mathbf{H}) = P \left( \sum_{k', d_{free}} \|Y(k) - \mathbf{H}_e \mathbf{G} \hat{X}(k)\|^2 \leq \sum_{k', d_{free}} \|Y(k) - \mathbf{H}_e \mathbf{G} \tilde{X}(k)\|^2 \right)$$

where  $\sum_{k', d_{free}}$  means that we only consider the  $d_{free}$  terms of the inequality for which  $\tilde{X}(k)$  and  $\hat{X}(k)$  are different. Consequently,

$$P(\underline{c} \rightarrow \hat{c} | \mathbf{H}) = P \left( \sum_{k', d_{free}} \omega_k \leq 0 \right)$$



(a) SDM vs GC for CC = (5,7)



(b) SDM vs GC for CC = (133,171)

Fig. 2. Simulation results: SDM vs the Golden code in a  $2 \times 2$  BICM system for a QPSK modulation with [5 7] encoder ( $d_{free} = 5$ ), and [133 171] encoder ( $d_{free} = 10$ )

$$\begin{aligned} \text{where } \omega &= \sum_{k', d_{free}} \omega_k \\ &= \sum_{k', d_{free}} \left\| \mathbf{H}_e \mathbf{G} \left( \hat{X}(k) - \tilde{X}(k) \right) + Z(k) \right\|^2 - \|Z(k)\|^2 \end{aligned}$$

In the following,  $D(k)$  denotes

$$D(k) = \hat{X}(k) - \tilde{X}(k)$$

Due to the bit interleaver, bits are decorrelated. Therefore,  $\omega$  is the sum of  $d_{free}$  independent Gaussian variable

$$\omega_k \sim \mathcal{N} \left( \|\mathbf{H}_e \mathbf{G} D(k)\|^2, 4N_0 \|\mathbf{H}_e \mathbf{G} D(k)\|^2 \right)$$

Consequently,  $\omega$  is a Gaussian variable with mean  $\sum_{k, d_{free}} \|\mathbf{H}_e \mathbf{G} D(k)\|^2$  and variance  $4N_0 \sum_{k, d_{free}} \|\mathbf{H}_e \mathbf{G} D(k)\|^2$ .

It follows that

$$P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}) \leq Q \left( \sqrt{\frac{\sum_{k', d_{free}} \|\mathbf{H}_e \mathbf{G} D(k)\|^2}{4N_0}} \right)$$

Using  $Q(x) \leq \exp(-\frac{x^2}{2})$ ,

$$P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}) \leq \prod_{k', d_{free}} \exp \left[ -\frac{\|\mathbf{H}_e \mathbf{G} D(k)\|^2}{8N_0} \right]$$

Averaging over all the channels, the PEP can be written as:

$$\begin{aligned} P(\underline{c} \rightarrow \hat{\underline{c}}) &\leq \mathbf{E}_{\mathbf{H}} \prod_{k', d_{free}} \exp \left[ -\frac{\|\mathbf{H}_e \mathbf{G} D(k)\|^2}{8N_0} \right] \\ &\leq \prod_{k', d_{free}} \exp \left[ -\frac{\mathbf{E}_{\mathbf{H}} (\|\mathbf{H}_e \mathbf{G} D(k)\|^2)}{8N_0} \right] \end{aligned}$$

Let  $C(k)$  denotes the  $2 \times 2$  matrix associated to the  $1 \times 4$

vectorized vector  $\mathbf{G}D(k)$ , such that  $\mathbf{G}D(k) = \text{vect}(C(k))$ . Then,

$$\|\mathbf{H}_e \mathbf{G} D(k)\|^2 = \|\mathbf{H}C(k)\|_F^2$$

where  $\|\cdot\|_F^2 = \text{tr}(\cdot)(\cdot)^H$  is the Frobenius norm. Thus,

$$PEP \leq \prod_{k', d_{free}} \exp \left[ -\frac{\mathbf{E}_{\mathbf{H}} (\|\mathbf{H}C(k)\|_F^2)}{8N_0} \right] \quad (8)$$

Following [4], the PEP is bounded by

$$\begin{aligned} PEP &\leq \prod_{k', d_{free}} \left[ \left( \prod_{i=1}^r \lambda_i \right)^{-n_r} \left( \frac{1}{8N_0} \right)^{-rn_r} \right] \\ &= \left[ \left( \prod_{i=1}^r \lambda_i \right)^{-n_r} \left( \frac{1}{8N_0} \right)^{-rn_r} \right]^{d_{free}} \quad (9) \end{aligned}$$

where  $\lambda_i$  are the eigen values of  $C(k)C(k)^H$  and  $r$  is the rank of this matrix.

From equation (9), we notice that the design criteria in [4] for uncoded MIMO systems still apply to the BICM-STBC case. The Golden code is also the optimal code for  $2 \times 2$  coded MIMO systems since it achieves the DMT by construction with full diversity order which is  $2n_r d_{free}$ . For the SDM case, the achieved diversity order is  $n_r d_{free}$ , i.e. half of the diversity order that can be experienced with the Golden code. However, when the binary code is robust enough, i.e.  $d_{free}$  is sufficiently large, the impact of diversity order will only be seen at high SNRs or low PERs. The additional diversity provided by the Golden code will only impact the system performance when  $d_{free}$  is relatively low.

### C. Numerical results

The performance of the Golden code versus SDM has been evaluated over a Rayleigh channel in terms of packet error rate (PER) versus SNR, for a packet length of 1000-bits. In the following, the SNR gain will be measured at a PER of  $10^{-2}$ . In order to assess our theoretical results, we compare the Golden

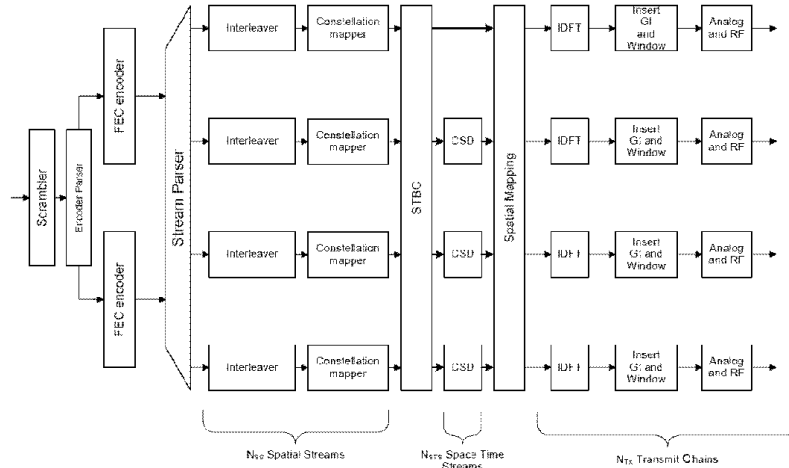


Fig. 3. Transmission Block Diagram for IEEE 802.11 n

code and SDM when concatenated to the convolutional codes  $CC_{10} = [133,171]$  and  $CC_5 = [5,7]$  with  $d_{free} = 10$  and 5 respectively. The PER performance is evaluated over a quasi-static Rayleigh channel in a  $2 \times 2$  MIMO configuration using QPSK constellation with coding rate  $\frac{1}{2}$ . Fig. 2(a) shows that the gain is 1.9 dB when using the low  $d_{free}$  convolutional code  $CC_5$ . This gain is significantly higher than the one provided by the GC when concatenated to  $CC_{10}$  with higher free distance Fig. 2(b). In this case, the 0.2 dB gain versus SDM confirms the impact of  $d_{free}$  on the gain provided by the Golden Code.

### III. ASSESSMENT OF THE GOLDEN CODE IN IEEE 802.11n

#### A. Presentation of the transmission chain of IEEE 802.11n

One major difference of IEEE 802.1n compared to other IEEE802.11x PHY layer architectures (e.g. IEEE802.11a) is the introduction of multiple transmit and multiple receive antenna concepts in order to exploit the Multiple Input Multiple Output (MIMO) channel properties. The transmission block diagram for IEEE 802.11n simulator is given in Fig. 3. The stream parser divides the output of the encoders into blocks that will be sent to different interleaver and mapping devices for rate increase compared to a single antenna system. Note that the number of convolutional encoders depends on the number of spatial streams. This spatial division multiplexing operation is optionally followed by space-time block coding for range increase. The constellation points from one spatial stream are spread into two space-time streams using one of the space-time block codes (STBC). Finally, spatial mapping can be optionally applied to map the resulting space-time streams to different transmit chains. Note that this last functional block is intended to perform spatial processing that is unknown at the receiver. The receiver will then perform the operations that are required for decoding of the space-time streams at the input of the spatial mapping block and that correspond to the content of the signaling field. This is enabled by applying this

spatial mapping block to the data, but also to the preambles, for appropriate channel estimation and data decoding. Finally OFDM modulation is applied prior transmission of the space-time streams to the analog and RF chains.

The data are encoded using the 802.11a rate 1/2 convolutional coder characterized by the polynomials 133 (octal) and 171 (octal). Six tail bits are required for the termination of the code. Two puncturing modes have already been specified for 802.11a in order to provide rate 2/3 and rate 3/4 coding schemes. The bit interleaver is used for frequency and spatial interleaving. Mapping operation is performed using one of the following the gray coded constellations: BPSK, QPSK, 16QAM and 64QAM. Spatial Division Multiplexing is used for all modes in which the number of transmit antennas is equal to the number of spatial streams. Robust transmission modes based on Space-Time Block Coding have also been defined. They consist in transmitting a number of spatial streams which is inferior to the number of transmit antennas. Then they are applicable in asymmetrical antenna configurations when the number of transmit antennas is superior to the number of receive antennas. The OFDM modulation for 20MHz bandwidth is based on a 64-point IFFT, with 52 data subcarriers and 4 pilots on subcarriers  $\pm 7$  and  $\pm 21$ .

#### B. Assessment of the Golden code in IEEE 802.11 n

The performance of the Golden code versus SDM is evaluated in the IEEE context in terms of packet error rate (PER) versus SNR, for a packet length of 1000-bits. In the following, SNR gain will be related to a PER of  $10^{-2}$ . Packet Error Rates in Fig.4 are evaluated over channel D [8] using QPSK constellation. This channel is characterized by a 50 ns rms delay spread and 18 taps, and then by significant frequency diversity. In the first scenario which refers to IEEE 802.11n without applying the convolutional code, the gain in terms of diversity and coding gain provided by the Golden code is very significant and leads to a 8dB gain compared to SDM.

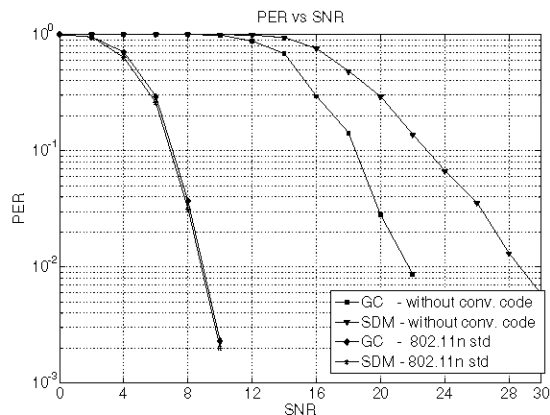


Fig. 4. SDM vs GC in 2 different cases for QPSK 1/2 - Channel model D

However, in the IEEE 802.11n context using  $R_c = 1/2$ , there is no additional gain at a  $PER = 10^{-2}$  due to the high diversity order achieved by the [133 171] rate 1/2 convolutional code.

#### IV. CONCLUSION AND PERSPECTIVES

In this paper, the performance of the Golden code compared to SDM (uncoded MIMO scheme) in a BICM-MIMO system and in IEEE 802.11n context is studied. We derive the expression of the pairwise error probability in a BICM-MIMO system and we find the maximal diversity order. We show that the Golden code is also optimal for BICM-MIMO system. However, the impact of the additional gain provided by the Golden code is reduced when using binary outer code with high free distance as in IEEE 802.11n standard.

Only upgrading the STBC block of 802.11n transmission chain does not allow to improve the performance of the coded system. A modification of the other blocks of the transmission

can be then envisaged. This can be performed by using for instance set partitioning at the mapper. Indeed, in [9], Hong et al. proposed to partition the Golden code in order to improve the coding gain by increasing the minimum determinant, and this work could be extended to BICM-MIMO systems.

#### V. ACKNOWLEDGMENT

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