Semi-Exhaustive Reduced-Complexity Recursive Block Decoding for MIMO Systems

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***Abstract*—In this work we propose a new recursive block decoding method for MIMO systems. A complexity reduction is obtained compared to existing methods by avoiding the exhaustive decoding on one block. We propose the use of a semi-exhaustive search using a list composed of the Maximum Likelihood (ML) solution and its neighborhoods. We show through error probability derivation that we can achieve the full diversity by fixing a judicious choice of the list size or equivalently a threshold metric. A desired diversity order can be obtained by changing the list size. Simulation results show the obtained performances and the complexity gain.**

***Index Terms*—MIMO systems, Recursive decoding, Block decoding, Diversity order, Reduced complexity.**

I. Introduction

Recent years witnessed a significant development of multiple-input multiple-output (MIMO) systems over scattering-rich wireless channels [1] for their ability to answer the increasing need for more reliability and data rate on wireless networks. Many decoders have been then adopted to retrieve signal streams sent over these systems with a good performance. However, the maximum likelihood decoders such as sphere decoder [2] or Schnorr Euchner [3] algorithms known to be optimal decoders require an exponential complexity in the number of antennas [4]. Improvements/Modifications of these decoders have been proposed to reduce their complexity at the possible cost of performance degradation providing, hence, a tradeoff between complexity and performance.

In the literature, recursive block decoding for Space-Time coded systems [5–10] is shown to slightly reduce the known ML decoding complexities. Taking advantage of the equivalent channel matrix form induced from the code structure, partitioned signal sets are decoded successively. The main issue of these decoders is the use, at one step, of exhaustive search over one block which increases the overall complexity.

A novel strategy is presented in this paper. Avoiding the exhaustive search in the first decoding step, the algorithm calculates the needed number of candidates that guarantees a fixed diversity order and SNR gain. In addition, this algorithm offers a significant computational complexity reduction compared to the above mentioned algorithms.

The remainder of this paper is organized as follows. In section II we describe the system model. In section III we review the state of the art on block decoding. The proposed recursive block decoding scheme is presented in section IV. Via the derivation of error probability, we analyze in section V the achieved diversity order. In section VI, we provide complexity and performance simulation results. We conclude and present future work in section VII.

II. System Model and Problem Statement

We consider a MIMO system with n_t transmit and n_r receive antennas using spatial multiplexing scheme. The complex-valued representation of the channel output is given by:

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{s}_c + \mathbf{w}_c \tag{1}$$

where $\mathbf{H}_c \in \mathbb{C}^{n_r \times n_t}$ denotes the channel matrix of elements drawn i.i.d. according to the distribution $\mathbb{CN}(0,1)$ and assumed perfectly known at the receiver. $\mathbf{w}_c \in \mathbb{C}^{n_r}$ is the additive white Gaussian noise of variance $\sigma^2 \mathbf{I}_{n_r}$ and $\mathbf{s}_c$ is the transmitted vector carved in a M-ary QAM signal constellation. We consider a symmetric system i.e $n_t = n_r$. In order to obtain a lattice representation of the channel output, we apply the complex-to-real transformation to get the real-valued system given by:

$$\mathbf{y}_{2n_r \times 1} = \mathbf{H}_{2n_r \times 2n_t} \mathbf{s}_{2n_t \times 1} + \mathbf{w}_{2n_r \times 1} \tag{2}$$

This system is to be considered in the decoding process. When a length-T Space-Time code is used, the channel output can be written in the same form of (1) with the equivalent channel matrix $\mathbf{H}_{eq}$ given by:

$$\mathbf{H}_{eq} = \mathbf{H}_c \mathbf{\Phi} \tag{3}$$

where $\mathbf{\Phi} \in \mathbb{C}^{n_t T \times n_t T}$ corresponds to the coding matrix of the underlying code [11]. For simplicity, given that both uncoded and coded schemes result in a same real-valued lattice representation, we consider in the remaining of this work the spatial multiplexing scheme. Let $n = 2n_t$.

A. ML Detection

In the coherent case where $\mathbf{H}$ is considered known at the receiver side, ML decoder finds the information vector $\mathbf{s}$ minimizing

$$\hat{\mathbf{s}} = \underset{\mathbf{s} \in \mathcal{A}^n}{\operatorname{argmin}} \parallel \mathbf{y} - \mathbf{H}\ \mathbf{s} \parallel^2 \tag{4}$$

where $\mathcal{A}$ represents the M-ary QAM constellation to which belong the real and imaginary parts of information symbols. This system can be resolved by using lattice decoders based on tree-search algorithms [2]. To get the tree structure, a QR decomposition is applied on the lattice generator matrix $\mathbf{H}$. Equation (4) is equivalent to:

$$\hat{\mathbf{s}} = \underset{\mathbf{s} \in \mathcal{A}^n}{\operatorname{argmin}} \parallel \mathbf{y}' - \mathbf{R}\ \mathbf{s} \parallel^2 \tag{5}$$

where $\mathbf{Q}$ is an orthogonal matrix, $\mathbf{R}$ an upper triangular one and $\mathbf{y}' = \mathbf{Q}^{\mathbf{t}}\mathbf{y}$.

B. Block Division

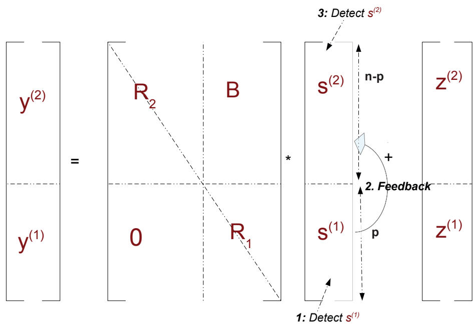

Fig. 1. Block division of the $\mathbf{R}$ matrix

We will consider a block division of matrix $\mathbf{R}$ to proceed to a recursive block decoding. The considered block division is depicted in Fig.1.

$\mathbf{R_1} \in \mathbb{R}^{p\times p}$ going to be decoded first is an upper triangular matrix. $\mathbf{R_2} \in \mathbb{R}^{n-p\times n-p}$ an upper triangular matrix is going to be decoded in the second stage. $\mathbf{B} \in \mathbb{R}^{n-p\times p}$ is a rectangular feedback matrix. $\mathbf{s^{(1)}}$ and $\mathbf{s^{(2)}}$ are the corresponding symbol vectors of size p and $n-p$ respectively. Based on this division, (5) can be rewritten as:

$$\begin{aligned}\hat{\mathbf{s}} &= \underset{(\mathbf{s^{(1)}},\mathbf{s^{(2)}})\in\mathcal{A}^p\times\mathcal{A}^{n-p}}{\operatorname{argmin}} \parallel \mathbf{y}' - \mathbf{R}\ \mathbf{s} \parallel^2 \\ &= \underset{(\mathbf{s^{(1)}},\mathbf{s^{(2)}})\in\mathcal{A}^p\times\mathcal{A}^{n-p}}{\operatorname{argmin}} \parallel \mathbf{y^{(2)}} - \mathbf{R_2}\ \mathbf{s^{(2)}} - \mathbf{B}\ \mathbf{s^{(1)}} \parallel^2 + \\ &\parallel \mathbf{y^{(1)}} - \mathbf{R_1}\ \mathbf{s^{(1)}} \parallel^2 \end{aligned} \tag{6}$$

III. Related Work

Reducing complexity while maintaining a good error performance and full diversity has been the object of many studies in the literature. We will focus on recursive signal set detection based works. Two main approaches are studied here.

The first approach is based on the division of the channel matrix in 2 blocks as in Fig.1. In [5] an ML decoding scheme is performed on the first block of size p, then a Zero Forcing Decision Feedback Equalizer (namely ZF-DFE) is applied to the remaining system given the output of the first ML decoding (i.e by subtracting the first ML output from the received signal). It was shown that this scheme is able to increase the diversity order for the worst sub-channel from 1 to p. An ordering scheme could be also applied to give the best decoding to the worst sub-channels, thus it is shown that an SNR gain equal to the number of transmitting antennas can be obtained.

The second approach, Space-Time coded systems oriented and compatible with sphere decoder [2], consists in splitting the received signal into $L \geq 2$ subsets each of cardinality λ. A conditional maximization of the likelihood function with respect to one signal set point given another is performed. Informally:

1) Exhaustive search for one sub-set.
2) Remove interference of all possible values of the first sub-set from remaining $L-1$ sub-sets.
3) Decode $L-1$ sub-sets with a ZF decoder for each decoded point of the first block.
4) Select optimal solution overall calculated solutions.

The choice of the signal set to decode first is crucial for the performance of the algorithm i.e to guarantee a maximal desired diversity order. Thus empirical [6–8] and analytical [9] set selection criteria on the equivalent channel matrix are derived. In [6] (and [7], [8]), authors examined the cases of Golden Code [12] (and 3×3, 4×4 perfect codes [13] and any $n\times n$ algebraic Space-Time code respectively). In these works, the main set selection criteria considered are the determinant of covariance matrices of the sub-channels. This quantity measures the instantaneous SNR of the corresponding linear system and thus should be large. Another criterion was also studied which is the condition number of this covariance matrix which measures the accuracy of the zero-forcing approximation and thus should be small. Then, the ratio of these quantities should be maximized. It was experimentally found that in the case of Golden Code, the sub-channel whose condition number is the smallest has the biggest determinant and thus a determinant based criterion is sufficient. In Perfect Code case, the condition number based criterion makes the performance slightly better and thus we can obviate the need to compute it too, taking into account the additional condition number computation complexity to be added in this case.

In [9], inspired from the above mentioned works, authors introduce two new low complexity decoders namely ACZF (Adaptive Conditional Zero-Forcing)and ACZF-SIC(Adaptive Conditional Zero-Forcing with Successive Interference Cancellation) where they give 2 equivalent sufficient conditions based on STBC characteristics to get full diversity with these decoders. One sufficient condition is the full rank of at least one of the L sub-matrices.

IV. The Proposed Recursive Block Decoding

In this section we present the main result of the paper which is semi-exhaustive recursive block decoding. In [9](Table 1), it is shown that for some ST codes, decoding complexity is very slightly reduced using the proposed recursive block decoder compared to known ML decoding complexities of these codes. This marginal gain is due to the exhaustive search performed in the first step (decoding of first block).

Our proposed decoder solves this issue by reducing the number of candidates kept in the first step compared to the exhaustive search. In addition, it offers a flexibility on the diversity order (impacting the overall complexity) by choosing a target diversity order less or equal to the full diversity imposed in the above mentioned works. The control of the diversity order is obtained through the choice of decoding parameters (like: block size, block order, list size or equivalently a stopping radius).

Parameters such as SNR and constellation size are also taken into account.
We divide the $\mathbf{R}$ matrix into 2 sub-blocks and we split, accordingly, the n real information symbols contained in s . Then, 4 steps are performed:

1) Generate a list containing the ML solution and some of its neighborhoods as an output of the decoding of the first block .
2) Subtract the interference of the decoded block (for each list point) from the remaining system.
3) ML or ZF-DFE decoding of the second block for each candidate of the list.
4) Select the solution that minimizes the overall ML metric with respect to (4)

For the first stage, we propose 2 possibilities to generate the list:

- by looking for all points inside a sphere centered on ML solution adapted for a decoding using Sphere Decoder.
- by looking to construct a list of fixed size adapted for a decoding using Stack Decoder.

The equivalence between a sphere radius r and the number of constellation points inside this sphere was studied in [14]. Let N_p be the number of lattice points contained in this sphere ($\mathbf{R_2}$ is the generator matrix of Λ) and N_e be the effective number of constellation points inside the sphere, we have:

$$r \approx \left(\frac{N_p \times vol(\Lambda)}{V_p}\right)^{\frac{1}{p}} \tag{7}$$

where: $vol(\Lambda) = \det(\mathbf{R_2})$ and V_p is the volume of a unit radius sphere in the real space $\mathbb{R}^p$, $V_p = \frac{\pi^{\frac{p}{2}}}{\frac{p}{2}!}$.
The effective number of lattice points in the list is $\alpha_{\mathcal{A}}{}^p \times N_p$. For example $\alpha_{4QAM} = \frac{3}{2}$, $\alpha_{16QAM} = \frac{3}{4}$ and $\alpha_{64QAM} = \frac{3}{8}$. Thus, $L = \left((\alpha_{\mathcal{A}} r)^p \frac{\pi^{\frac{p}{2}}}{\frac{p}{2}!}\right)/|\det \mathbf{R_2}|$.
An overview of the whole decoding scheme with an ML decoder in the the second phase is depicted in Fig.2.

A modified version of a sequential decoder, for example the SB-Stack decoder [15], could be used to generate these candidates where first the optimal solution of the system $\mathbf{y_2} = \mathbf{R_2}\ \mathbf{s_2} + \mathbf{z_2}$ is found, then all the near-ML solution lying inside the sphere of radius r are generated and stored to be expanded in the second decoding stage. As depicted in Fig.2, these branches represent the potential candidates among which the transmitted vector could be found with high probability.

V. Diversity Order Analysis

In this section, we derive an upper bound of the Frame Error Rate P_{ef}. Note that at high SNR regime, a frame error is caused, with high probability, by an error on one symbol. Thus,the Symbol Error rate P_{es} can be approximated by $P_{es} \approxeq P_{ef}/n$
Notations:
Denote by:

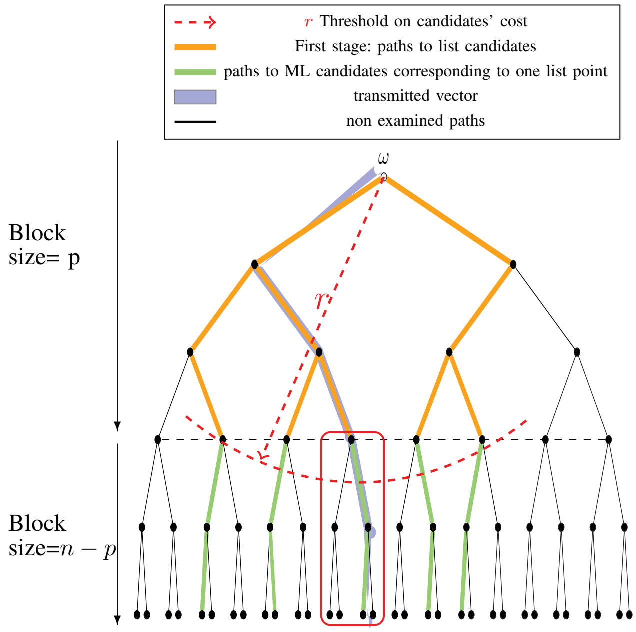

Fig. 2. Tree-based Decoding scheme overview

- $\mathbf{s} = \left[\mathbf{s^{(2)}}, \mathbf{s^{(1)}}\right]$ the transmitted vector, as explained in paragraph II-B where $\mathbf{s^{(2)}} = [s_n, s_{n-1} \dots s_{p+1}]$ and $\mathbf{s^{(1)}} = [s_p, s_{p-1}, \dots s_1]$.
- $\hat{\mathbf{s}} = \left[\widehat{\mathbf{s^{(2)}}}, \widehat{\mathbf{s^{(1)}}}\right]$ the output of the decoder.

Also, denote by:

- $\tilde{s}$ the event that $\mathbf{s}$ was visited during the search i.e $\mathbf{s}$ belongs to the set $\mathcal{I}$ of full paths metric-compared at the end of the second stage.
- $\widetilde{s^{(i)}}$ the event that $\mathbf{s^{(i)}}$ was visited during the search, $i = \{1, 2\}$.
- $\tilde{s}^c$ and $\widetilde{s^{(i)}}^c$ the opposite events of $\tilde{s}$ and $\widetilde{s^{(i)}}$ respectively.

Then, the Frame Error Rate $P_{ef} = \Pr(\mathbf{s} \neq \hat{\mathbf{s}})$ could be written, using conditional probability rule, as:

$$\begin{aligned} P_{ef} &= \Pr(\mathbf{s} \neq \hat{\mathbf{s}} \cap \tilde{s}) + \Pr(\mathbf{s} \neq \hat{\mathbf{s}} \cap \tilde{s}^c) \\ &= \Pr(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}) \Pr(\tilde{s}) + \underbrace{\Pr(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}^c)}_{=1} \Pr(\tilde{s}^c). \\ &= \Pr\left(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}\right) \Pr(\tilde{s}) + \Pr(\tilde{s}^c). \end{aligned} \tag{8}$$

It is clear that $\lim_{\rho \to \infty} \Pr(\tilde{s}) = 1$ where ρ is the instantaneous SNR.
We start by deriving $Pr\left(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}\right)$. It is the probability that, after having built our set $\mathcal{I}$ of couples $\left(\widetilde{s'}^{(1)}, \widetilde{s'}^{(2)}\right)$ and compared their global metric with respect to the system in (6), $\mathbf{s}$ doesn't have the smallest metric. Thus, using union bound

(UB):

$$\Pr\left(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}\right) = \mathbb{E}_{\mathbf{R}}\mathbb{E}_{\mathbf{s}} \sum_{\substack{\mathbf{s}' \neq \mathbf{s} \\ \mathbf{s}' \in \mathcal{I}}} \Pr\left(\left|\mathbf{y} - \mathbf{R}\mathbf{s}'\right|^2 \leq \left|\mathbf{y} - \mathbf{R}\mathbf{s}\right|^2\right) \tag{9}$$

We know that:

$$\Pr\left(\left|\mathbf{y} - \mathbf{R}\mathbf{s}'\right|^2 < \left|\mathbf{y} - \mathbf{R}\mathbf{s}\right|^2\right) \leq \mathcal{Q}\left(\sqrt{\frac{\left|\mathbf{R}\left(\mathbf{s} - \mathbf{s}'\right)\right|^2}{2\sigma^2}}\right) \tag{10}$$

$\mathbf{R}$ entries $|r_{i,i}|^2$ are distributed according to $\chi^2(2(n-i+1))$ [16] for $i = 1, \ldots, n$ with a maximum degree of freedom=$2n$. Using Chernoff bound for the $\mathcal{Q}$-function in (10), we obtain that:

$$\begin{aligned} \mathbb{E}_{\mathbf{R}}\mathcal{Q}\left(\sqrt{\frac{\left|\mathbf{R}\left(\mathbf{s} - \mathbf{s}'\right)\right|^2}{2\sigma^2}}\right) &\leq \mathbb{E}_{\mathbf{R}} \exp\left(-\frac{\left|\mathbf{R}\left(\mathbf{s} - \mathbf{s}'\right)\right|^2}{4\sigma^2}\right) \\ &\leq \frac{1}{\left(1 + \frac{|\mathbf{s}-\mathbf{s}'|^2}{4\sigma^2}\right)^n} \leq \frac{1}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \end{aligned} \tag{11}$$

Then,

$$\begin{aligned} \Pr\left(\mathbf{s} \neq \hat{\mathbf{s}} \mid \tilde{s}\right) &\leq \mathbb{E}_{\mathbf{s}} \sum_{\substack{\mathbf{s}' \neq \mathbf{s} \\ \mathbf{s}' \in \mathcal{I}}} \frac{1}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \\ &\leq \frac{|\mathcal{I}|}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} = \beta\rho^{-n} \end{aligned} \tag{12}$$

where d_{min} is the Euclidean distance between nearest neighbors in $\mathcal{A}$ and β a positive constant.
We derive now $\Pr(\tilde{s})$:

$$\begin{aligned} \Pr(\tilde{s}) &= \Pr\left(\tilde{s} \cap \widetilde{s^{(1)}}\right) + \underbrace{\Pr\left(\tilde{s} \cap \widetilde{s^{(1)}}^c\right)}_{=0} \\ &= \Pr\left(\tilde{s} \mid \widetilde{s^{(1)}}\right) \Pr\left(\widetilde{s^{(1)}}\right). \end{aligned}$$

Given that $\tilde{s} = \widetilde{s^{(1)}} \cap \widetilde{s^{(2)}}$, then $\Pr\left(\tilde{s} \mid \widetilde{s^{(1)}}\right) = \Pr\left(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}}\right)$.
Thus:

$$\Pr\left(\tilde{s}\right) = \Pr\left(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}}\right) \Pr\left(\widetilde{s^{(1)}}\right). \tag{13}$$

We derive $\Pr(\widetilde{s^{(1)}}^c) = 1 - \Pr(\widetilde{s^{(1)}})$. Recall that $\Pr(\widetilde{s^{(1)}}^c)$ is the probability that the metric associated to $\mathbf{s^{(1)}}$, with respect to the 1^{st} system in (6), falls over a certain fixed threshold r.

$$\begin{aligned} \Pr(\widetilde{s^{(1)}}^c) &= \mathbb{E}_{\mathbf{R_1}} \Pr(\mathbf{s^{(1)}}) \sum_{s_1 \in \mathcal{A}^p} \Pr(|\mathbf{y^{(1)}} - \mathbf{R_1}\mathbf{s^{(1)}}|^2 \geq r^2) \\ &= \Pr(|\mathbf{z^{(1)}}|^2 \geq r^2). \end{aligned} \tag{14}$$

Since $\frac{|\mathbf{z^{(1)}}|^2}{\sigma^2}$ is distributed according to the $\chi^2\left(p\right)$, then:

$$\begin{aligned} \Pr(\widetilde{s^{(1)}}^c) &= \Pr\left(\frac{\left|\mathbf{z}^{(1)}\right|^2}{\sigma^2} \geq \frac{r^2}{\sigma^2}\right) \\ &= \int_{\frac{r^2}{\sigma^2}}^{\infty} f(x)dx = \frac{\Gamma(\frac{p}{2}, \frac{r^2}{2\sigma^2})}{\Gamma(\frac{p}{2})} \end{aligned} \tag{15}$$

where f is the pdf of $\chi^2\left(p\right)$ and $\frac{\Gamma(a,x)}{\Gamma(a)}$ is the normalized upper Gamma function.

It remains now only the second term in (13): $\Pr(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}})$. Having kept a set $\mathcal{I}_1(|\mathcal{I}_1| < |\mathcal{A}|^p)$ of p-length paths for next decoding stage, this quantity depends on the decoding scheme that it is going to be used in the decoding of the second system $\mathbf{y^{(2)}} = \mathbf{R_2}\ \mathbf{s^{(2)}} + \mathbf{B}\mathbf{s^{(1)}} + \mathbf{z^{(2)}}$ in (6) (after a $|\mathcal{I}_1|$ times SIC operations).
We propose to study in V-A the case of an ML decoder chosen to be Sphere Decoder (SD) and in V-B the case of sub-optimal decoder chosen to be Zero-Forcing Decision Feedback Equalizer (ZF-DFE).

A. FER Analysis for ML Decoding in the 2^{nd} Stage

In this section, we provide the analytical proof that any diversity order $\kappa \in [1, \ldots n]$ is achievable.
For ease of calculations, we derive $\Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right) = 1 - \Pr\left(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}}\right)$.
$\Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right)$ is the probability that, after a SIC operation of $\mathbf{s^{(1)}}$, $\mathbf{s^{(2)}}$ is not the chosen output of the ML decoder i.e doesn't have the smallest metric among all its neighbors. Then, using the Union Bound (UB):

$$\begin{aligned} \Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right) \leq \mathbb{E}_{\mathbf{R_2}}\mathbb{E}_{\mathbf{s^{(2)}}} \sum_{\mathbf{s'^{(2)}} \neq \mathbf{s^{(2)}}} &\Pr(|\mathbf{y^{(2)}} - \mathbf{R_2}\mathbf{s'^{(2)}}|^2 \\ &< |\mathbf{y^{(2)}} - \mathbf{R_2}\mathbf{s'^{(2)}}|^2) \end{aligned} \tag{16}$$

Similarly to (11), $\mathbf{R}$ entry $|r_{n,n}|^2$ is distributed according to $\chi^2(2n)$. Then:

$$\begin{aligned} &\mathbb{E}_{\mathbf{R_2}}\left(\Pr\left(|\mathbf{y^{(2)}} - \mathbf{R_2}\mathbf{s'^{(2)}}|^2 < |\mathbf{y^{(2)}} - \mathbf{R_2}\mathbf{s'^{(2)}}|^2\right)\right) \\ &\leq \frac{1}{\left(1 + \frac{|\mathbf{s'^{(2)}} - \mathbf{s^{(2)}}|^2}{4\sigma^2}\right)^n} \leq \frac{1}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \end{aligned} \tag{17}$$

Hence (16) could be written as:

$$\begin{aligned} \Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right) &\leq \mathbb{E}_{\mathbf{s^{(2)}}} \sum_{\mathbf{s'^{(2)}} \neq \mathbf{s^{(2)}}} \frac{1}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \\ &\leq \frac{|\mathcal{A}|^{n-p}}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \end{aligned} \tag{18}$$

Now from (13),

$$\begin{aligned}\Pr\left(\widetilde{s}^c\right) &= 1 - \Pr\left(\widetilde{s}\right)\\ &= 1 - \left(1 - \Pr\left(\widetilde{s^{(1)}}^c\right)\right)\left(1 - \Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right)\right)\\ &\approx \Pr\left(\widetilde{s^{(1)}}^c\right) + \Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right) \qquad (19)\end{aligned}$$

since $\Pr\left(\widetilde{s^{(1)}}^c\right)$ and $\Pr\left(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}}\right)$ are small at high SNR. Combining (12), (19), (18) and (15), the FER in (8) is upper-bounded by:

$$P_{ef} \leq \frac{\Gamma(\frac{p}{2}, \frac{r^2}{2\sigma^2})}{\Gamma(\frac{p}{2})} + \frac{|\mathcal{I}| + |\mathcal{A}|^{n-p}}{\left(1 + \frac{d_{min}^2}{4\sigma^2}\right)^n} \qquad (20)$$

Equation (20) shows that the diversity order that could be achieved by this decoding scheme is controlled by the first term given that the second one achieves full diversity. Therefore, to guarantee an overall diversity order of at least κ, the first term (function of the block size p, the noise term σ^2 and the metric threshold r) should decrease at the order of $\sigma^{2\kappa}$. This goes back, for a given fixed high SNR (or small σ^2) and block size p, to find the minimum threshold r that satisfies:

$$\frac{\Gamma(\frac{p}{2}, \frac{r^2}{2\sigma^2})}{\Gamma(\frac{p}{2})} \leq \delta\sigma^{2\kappa} \qquad (21)$$

for some positive constant δ that controls the SNR gain. This inequality on r is solved numerically in simulations with a margin of error as small as possible.

B. FER Analysis for ZF-DFE Decoding in the 2^{nd} Stage

In this section, we show that the overall diversity order provided by this decoding scheme for block decoding where $n - p \geq 2$ is limited by the sub-optimal decoding in the second stage i.e $\kappa_{max} = 1$.

Same as in V-A, we derive $\Pr\left(\widetilde{s^{(2)}}^c \mid \widetilde{s^{(1)}}\right) = 1 - \Pr\left(\widetilde{s^{(2)}} \mid \widetilde{s^{(1)}}\right)$ in the case where a ZF-DFE decode with no channel ordering is used for the 2^{nd} decoding stage given that $\mathbf{s}^{(1)}$ was visited(i.e decoding inside the framed sub-tree in Fig.2). Now the ZF-DFE decoder is looking for the estimate $\widehat{\mathbf{s}^{(2)}}$ with respect to the following system:

$$\mathbf{y}^{(2)} = \mathbf{R_2 s}^{(2)} + \mathbf{B s}^{(1)} + \mathbf{z}^{(2)} \qquad (22)$$

After SIC operation using the transmitted sub-vector $s^{(1)}$ (i.e no error propagation), the system is rewritten as:

$$\mathbf{y}'^{(2)} = \mathbf{y}^{(2)} - \mathbf{B s}^{(1)} = \mathbf{R_2 s}^{(2)} + \mathbf{z}^{(2)} \qquad (23)$$

Given assumptions on noise statistics and that no channel ordering is performed, ZF-DFE decoder, applied on such system, is known to provide a maximum diversity order of $\kappa_{max} = 1$ which controls then the overall diversity order (even if an exhaustive search in the first stage is performed).

Note that if $n - p = 1$ i.e only one symbol left to detect, the ZF-DFE decoder coincides with ML decoder and thus diversity order κ follows the same rule as in previous section.

VI. Simulation Results

The proposed decoder has been validated by numerical simulations considering spatial multiplexing for $n_t = n_r = 4$ using a 4-QAM modulation. Block dimensions are $p = n - p = 4$ ($n = 8$ in real-valued system). We provide here numerical results evaluating the Symbol Error Rate SER and the average decoding complexity(computed as the overall number of multiplications) averaging over 2×10^6 channel realizations.

We compare diversity order in Fig.3 and complexity in Fig.4 of Sphere Decoder with 4 possible configurations of our decoder scheme by varying the minimum target diversity order κ and the SNR gain factor δ.

In the first scenario (black line), we set the target diversity $\kappa = 4$ (i.e full diversity). We observe that this diversity order is indeed achieved with a noticeable complexity reduction compared to the SD.

In the second and third scenario, we set the minimum target diversity $\kappa = 2$ with 2 variants for SNR gain factor δ. We validate in Fig.3 that SER curves (blue and magenta lines) have a same slope of 2 with different SNR gains. The last scenario(in green) depicts the case where the minimum target diversity order is set to $\kappa = 1$.

We can observe that at low SNR regime, the second,third and fourth scenarios (blue, magenta and green curves) have higher complexity than the first considered scenario. This behavior is due to the choice, at this regime, of a variable delta function of SNR in the first scenario, which seems to be the best way for the choice of delta. This behavior could be explained by the fact that for small values of ρ, $\delta\rho^{-n} \to 1$ which gives a too small radius (due to properties of normalized upper gamma function), thus no lattice point could be found, causing the algorithm to restart and increase complexity.

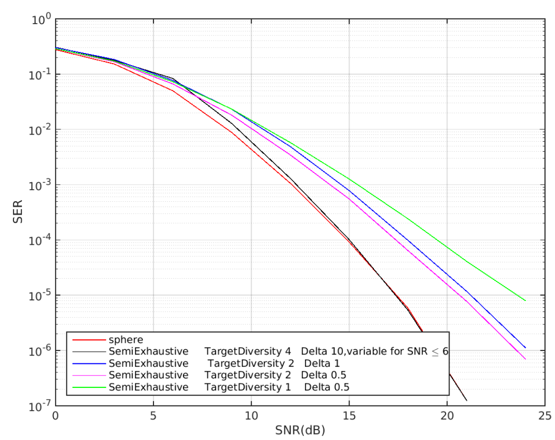

Fig. 3. Symbol Error Rate for $n_t = n_r = 4$ using 4-QAM constellation and spatial multiplexing

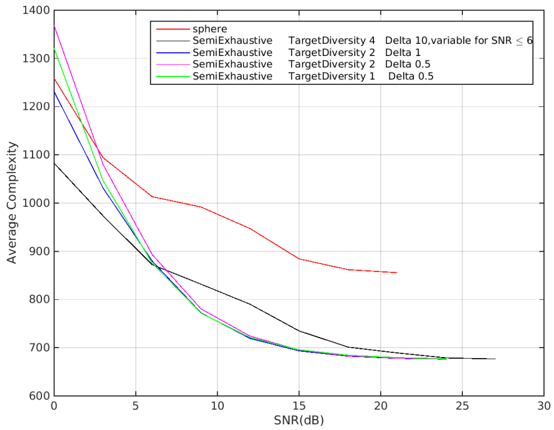

Fig. 4. Total computational complexity for $n_t = n_r = 4$ using 4-QAM constellation and spatial multiplexing

VII. Conclusion

This work was dedicated to semi-exhaustive recursive block decoding implementable in linear communication systems including MIMO systems. A complexity reduction coupled with a flexibility on desired diversity order, between minimum and full diversity for a given system is achieved. In this work, the case of 2 blocks of the same size was studied, in future work we are considering, among others, the case of division into more than 2 sub-blocks with different sizes. Also, we propose to combine our algorithm based on ZF-DFE decoder in 2^{nd} stage with block division and ordering based on the determinant of the covariance matrices of blocks. Then, having as input the $\mathbf{R}$, for all block division sizes $p \in [2, .., n-1]$, the algorithm compares the determinants of $\mathbf{G_1} = \mathbf{R_1}^H \mathbf{R_1}$ and $\mathbf{G_2} = \mathbf{R_2}^H \mathbf{R_2}$ and chooses the block having the largest determinant to be the 1^{st} block to be decoded. It was shown in prior art that based on this criteria, the ZF-DFE is able to achieve full diversity.
Simulations are reported for an extended version of this paper.

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