Space-Time Codes for Fiber Communications: Coding Gain and Experimental Validation

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Abstract—**Dual polarization coherent systems are considered to be an attractive solution for optical fiber transmissions beyond 100Gb/s. Polarization multiplexing (PolMux) doubles the channel capacity and the spectral efficiency by transmitting two signals on the orthogonal polarization states. However optical polarization-dependent impairments, such as polarization mode dispersion (PMD) or polarization dependent loss (PDL) in optical fibers, may induce severe performances degradation. As PolMux systems can be seen as 2×2 MIMO channels, Space-Time codes can be used to enhance the transmission performance. The performances of the Golden code and the Silver code, which are the two best Space-Time codes in wireless communications, are evaluated in the case of optical PolMux OFDM systems by numerical simulations and validated by experiments. We show that Space-Time coding are inefficient against dispersion effects such as PMD but can dramatically mitigate PDL effects. Moreover, the obtained results show that, unlike in wireless communications, the Silver code outperforms the Golden code.**

Index Terms— **Optical fiber communication, OFDM modulation, Optical polarization, Modulation coding**

I. INTRODUCTION

The rapid growth of global IP traffic has put more and more capacity demand on the transport networks [1]. The actual 10Gb/s-based wavelength division multiplexing (WDM) technologies allow maximum capacity of 0.5-1Tb/s per fiber. The 100Gb/s Ethernet (100GbE) is considered to become the next generation Ethernet standard for IP networks. Modulation formats based on direct-detection result in limited chromatic dispersion and PMD tolerance. New approaches based on coherent detection combined with digital signal processing (DSP) appear currently the more promising solutions. They allow higher spectral efficiency [2] and compensation of linear impairments in the electrical domain [3]. Polarization division multiplexing is an attractive solution to double the spectral efficiency. A polarization diversity receiver is required to detect the received PolMux optical signal. After that, a DSP unit performs polarization recovery, equalization, carrier recovery and symbol decoding. First real-time PolMux QPSK solutions have been reported at 40Gb/s [4] and 100Gb/s [5], showing a large chromatic dispersion and PMD tolerance. Orthogonal Frequency Division Multiplexing (OFDM) is presented as an alternative solution to single-carrier QPSK format because of its high resilience to chromatic dispersion and PMD, and its reduced DSP complexity also (1Tap 2x2 MIMO equalizer) [6,7].

The most important polarization dependent impairments are PMD and PDL respectively. PMD is caused by the different group velocities between the propagating polarization modes. PMD can be mitigated by efficient equalization techniques [3] or by using OFDM with a well-dimensioned cyclic prefix [7]. PDL is a fading effect introduced by in-line optical components (isolators, amplifiers, etc) which attenuate in a different way and randomly the two polarization states. Hence, PDL induces polarization dependent power fluctuations resulting in unequal optical signal-to-noise ratio (OSNR) on each received polarization. Note that PDL cannot be efficiently mitigated by digital equalization techniques.

Space-Time (ST) codes have been introduced in wireless communications in order to exploit all the degrees of freedom of a MIMO channel [8]. They can be adapted to optical communications and sometimes are referred in literature as Polarization-Time (PT) codes [9,10]. The Golden code [11] and the Silver code [12] were proven to be the best codes on 2x2 Rayleigh fading channels.

In this paper, we will inquire into the benefit of using ST codes for optical OFDM transmission systems. We will start by presenting the typical structure of a 2x2 MIMO OFDM system, followed by a model description of the optical fiber channel. Then the PT coding will be presented. We will show that PT coding techniques are unable to manage dispersion impairments but can very efficiently mitigate the performances degradation due to random fading induced by PDL. Finally, performance evaluation of Golden code and Silver code will be presented through numerical simulations and experiments. Unlike in wireless communications, the Silver code performs better than the Golden code. The obtained results encourage us to search for more appropriate PT codes for optical fiber channels.

II. POLARIZATION-MULTIPLEXED OFDM SYSTEMS

A. OFDM system

The OFDM format consists of multiplexing the emitted symbols over multiple independently modulated orthogonal subcarriers. The modulated symbols are assigned in the frequency domain and the signal is converted to the time domain by an inverse FFT. In order to manage dispersion effects (chromatic dispersion and PMD), a cyclic prefix is inserted before each OFDM symbol. It acts as a guard interval to avoid inter-symbol interference (ISI). In the case of PolMux systems, the

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signal is transmitted and received on two orthogonal polarizations. A good description of optical PolMux OFDM systems can be found in [6,7].

The received signal for a linear optical channel can be expressed in the frequency domain as:

$$\boldsymbol{Y}_k = \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{N}_k \tag{1}$$

where $\boldsymbol{H}_k$ is the 2x2 channel transfer matrix and $\boldsymbol{X}_k$, $\boldsymbol{Y}_k$, $\boldsymbol{N}_k$ are respectively the transmitted symbol vector, the received signal vector and the noise on the k^{th} subcarrier. As the dominant noise in a linear optical channel is the amplified spontaneous noise (ASE) induced by optical amplifiers, N_k can be modeled as an additive white Gaussian noise. The channel transfer matrix can be estimated with a training sequence and is supposed known at the receiver. The information symbols can be recovered optimally at the receiver by maximum-likelihood (ML) decoding:

$$\hat{\boldsymbol{X}}_k = \underset{X}{arg\,min} \left\| \boldsymbol{Y}_k - \boldsymbol{H}_k \boldsymbol{X}_k \right\|^2 \tag{2}$$

if a QPSK modulation is considered for the different subcarriers, ML decoding can be performed by an exhaustive research with reasonable complexity.

The capacity of this channel per subcarrier can be expressed as in [8]:

$$C_k = \sum_{i=1}^{2} E_{\lambda_i} \left\{ log_2 \left(1 + \frac{\rho}{2} \lambda_i^2 \right) \right\} \tag{3}$$

where ρ is the signal to noise ratio for the sub-carrier and λ_i are the singular values of $\boldsymbol{H}_k$. The total capacity of the whole system is the sum of the capacities of all sub-carriers.

B. *Optical channel model*

After propagation through optical fiber, the received signal is affected by various physical impairments such as chromatic dispersion, PMD, PDL and nonlinear effects. In this paper, we focus on polarization dependent effects. PMD and PDL are linear effects and can be modeled using a 2x2 channel transfer function matrix.

1) *Polarization mode dispersion*: The slight geometrical imperfections and anisotropic stresses cause modal birefringence. As the birefringence is not constant along the fiber but changes randomly, the accumulated differential group delay (DGD) varies in time. This phenomenon is referred as PMD. The total PMD is typically considered as a concatenation of PMD elements $\mathbf{H}_{\mathrm{PMD}}$ such as [13]:

$$\boldsymbol{H}_{PMD}(\omega) = \boldsymbol{R}_\alpha \begin{bmatrix} exp\left(i\frac{\omega\Delta\tau}{2} \right) & 0 \\ 0 & exp\left(-i\frac{\omega\Delta\tau}{2} \right) \end{bmatrix} \boldsymbol{R}_\beta \tag{4}$$

where $\boldsymbol{R}_\alpha$ and $\boldsymbol{R}_\beta$ are random rotation matrices - the angles are chosen from a uniform distribution with values in $[0,2\pi]$ - and the matrix in the middle represents the frequency dependent delay with $\Delta\tau$ being the DGD over one PMD element. We notice that when only PMD is considered, the transfer matrix is unitary ($\boldsymbol{H}_{PMD}\boldsymbol{H}_{PMD}^H = \boldsymbol{I}$). Hence, the capacity per subcarrier is not reduced and is equivalent to the capacity of two parallel Gaussian channels. Therefore, ST coding techniques cannot bring any improvement to a system with pure dispersive impairments.

2) *Polarization dependent loss*: PDL is a fading effect introduced by the in-line optical components. Signals with distinct polarization states are not identically attenuated (i.e. see Fig. 1). As a long-haul optical transmission system includes a large number of in-line components, it can result in a significant amount of total PDL. Each PDL element can be represented by the transfer matrix:

$$\boldsymbol{H}_{PDL} = \boldsymbol{R}_\alpha \begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & \sqrt{1+\gamma} \end{bmatrix} \boldsymbol{R}_\beta \tag{5}$$

The attenuation parameter γ is related to the PDL noted Γ_{dB} by:

$$\Gamma_{dB} = 10\ log_{10} \frac{1-\gamma}{1+\gamma} \tag{6}$$

It has been shown that PDL in long-haul fiber systems is a stochastic process following a typical Gaussian distribution [14]. Therefore, we consider in our work a PDL model as in (5) in which Γ_{dB} is a standard deviation of a zero-mean Gaussian distribution (i.e. *N(0,PDL)*).

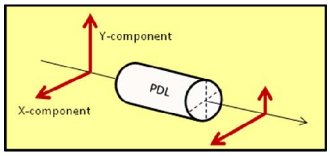

Fig. 1 Principle of PDL when the axes of the optical element are parallel to the polarizations of the input signal.

III. POLARIZATION-TIME CODING

A. *Polarization-Time codes in optical transmission systems*

OFDM ensures a non-dispersive channel model in the frequency domain. Thus, PT coding can be applied as in Fig. 2.

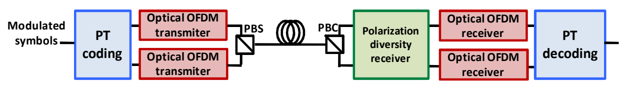

Fig. 2 Scheme of a 2x2 OFDM transmission with Polarization-Time coding.

Systems combining OFDM format and PT coding have been proposed in literature. PT coding consists in sending a linear combination of modulated symbols on each polarization *(Pol_1,Pol_2)* during several channel uses. On a 2x2 MIMO channel, to achieve full rate and full diversity, the encoded symbols are sent on the 2 polarizations *(Pol_1,Pol_2)* during 2 symbol times *(T_1,T_2)*:

$$\boldsymbol{X} = \begin{bmatrix} X_{Pol_1,T_1} & X_{Pol_1,T_2} \\ X_{Pol_2,T_1} & X_{Pol_2,T_2} \end{bmatrix} \tag{7}$$

Many ST codes have been designed for 2x2 channels. We will focus especially on the Golden and Silver codes.

1) The Golden code: it was shown that the Golden code has the best performances on a 2x2 MIMO Rayleigh fading channel [11]. The codeword matrix of the Golden code is:

$$\boldsymbol{X}_G = \begin{bmatrix} \alpha(S_1+\theta S_2) & \alpha(S_3+\theta S_4) \\ i\bar{\alpha}(S_3+\bar{\theta} S_4) & \bar{\alpha}(S_1+\bar{\theta} S_2) \end{bmatrix} \tag{8}$$

where S_1, S_2, S_3, S_4 are 4 QPSK symbols, $\theta = (1+\sqrt{5})/2$, $\bar{\theta} = (1-\sqrt{5})/2$, $\alpha = 1+i-i\theta$, and $\bar{\alpha} = 1+i-i\bar{\theta}$. The codeword matrix of the Golden code is full rank which ensures the maximum diversity. It achieves a full rate of 2 symbols by channel use because 4 symbols are transmitted during 2 symbol times.

2) The Silver code: the Silver code performance is very close to the Golden code performance but has also the advantage of reduced decoding complexity due to its particular structure [12]. The codeword matrix of the Silver code is:

$$\boldsymbol{X}_S = \begin{bmatrix} S_1+Z_3 & -S_2^*-Z_4^* \\ S_2-Z_4 & S_1^*-Z_3^* \end{bmatrix}$$

$$\begin{bmatrix} Z_3 \\ Z_4 \end{bmatrix} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1+i & -1+2i \\ 1+2i & 1-i \end{bmatrix} \begin{bmatrix} S_3 \\ S_4 \end{bmatrix} \tag{9}$$

where S_1, S_2, S_3, S_4 are 4 QPSK symbols.

B. Design criterion

The Golden and Silver codes were designed in order to minimize the upper bound of the pair-wise error probability on a Rayleigh fading channel. It results in a coding gain proportional to the minimum determinant of the difference of two codeword matrices which is equal to *1/7* for the Silver code and *1/5* for the Golden code. Thus the Golden code outperforms the Silver code. However the optical fiber channel is different from the wireless channel and the design of PT codes is not based on the same criteria. The error probability is a function of the Euclidian distance between pairs of codewords. The transfer matrix $\boldsymbol{H}_{PDL}$ modifies the codeword constellation and reduces the distance between codewords, which increases the error probability. The minimum distance d_{min} corresponds to the smallest distance between a pair of constellation points in the re-normalized space (re-normalized by PDL):

$$d_{min} = \min_{X_1, X_2} \left\| \boldsymbol{H}_{PDL}(\boldsymbol{X}_1 - \boldsymbol{X}_2) \right\|^2 \tag{10}$$

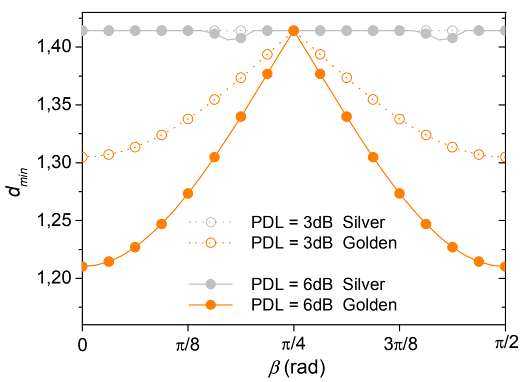

Fig. 3. Minimum distance of PT codes depending on the rotation angle β of the transfer matrix $\boldsymbol{H}_{PDL}$ for a QPSK constellation.

The PT code having the best performances in presence of PDL is the one having the highest minimum distance d_{min}. By examining (5), we notice that for each PDL value, the transfer matrix $\boldsymbol{H}_{PDL}$ is function of the random angles α and β. However, the minimum distance of the code is only function of β because $\boldsymbol{R}_\alpha$ is a rotation matrix that does not change d_{min}. In Fig. 3, the minimal distances d_{min} of the Golden code and Silver code are plotted in function of the rotation angle β. We can observe that for a PDL of 3dB, the minimal distance is constant which is not the case for the Golden code. When PDL is increased to 6dB, Silver code is still quite insensitive to the variation of the rotation angle. The minimum distance d_{min} shows a π/2 periodicity. Unlike 2x2 MIMO Rayleigh fading channel, the Silver code is the best PT code to mitigate fading in an optical channel.

IV. SIMULATION RESULTS

We consider the OFDM system presented in Fig. 2 where a QPSK format is used. Fig. 4 shows the BER evolution versus SNR for a PDL of 3dB. We can clearly observe the efficiency of PT coding against PDL. Without PT codes, the SNR degradation induced by PDL is about 1dB at BER=10^{-3}. Using PT codes, the SNR penalty is only 0.2dB with the Golden code. We also see that the Silver code mitigates almost all the PDL impairments (i.e. BER very close to the case without PDL) [15]. These codes do not introduce any spectral efficiency penalties compared to the uncoded case as they are by construction redundancy free. Note that these results have been confirmed by E. Meron et al. [16]. The SNR penalty at BER=10^{-3} has been evaluated for

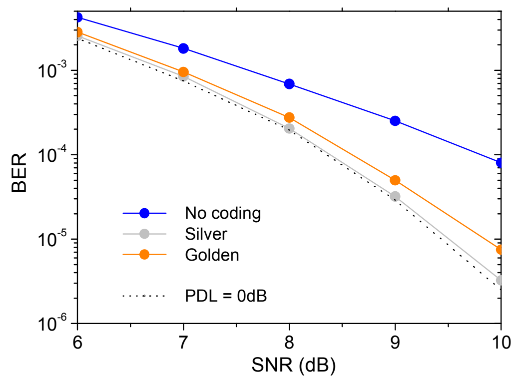

Fig. 4 Performance of Polarization-Time coding (PDL = 3dB).

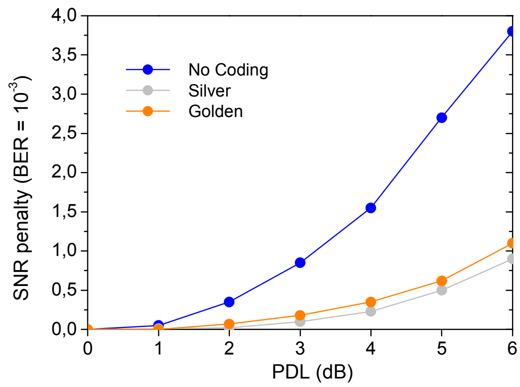

Fig. 5 SNR penalty induced by PDL at BER = 10^{-3}.

different PDL values as shown on Fig. 5. There is a penalty of about 3.8dB for a 6dB PDL. Using the Golden or Silver codes, the penalties are reduced to only 1dB which correspond to a ~3dB coding gain.

BER performances of combining PT coding with FEC have been also investigated. We have used a lattice LDPC code *C*(10470,9074). In Fig. 6, we show coding gain of the Silver code for a PDL of 6dB (with and without FEC). There is a coding gain of ~7dB at BER of 10^{-5} using FEC and Silver code compared to the uncoded case. The FEC brings 4.5dB and the PT code 2.3dB. There is less than 1dB between the case with and without PDL when FEC and PT-coding are used simultaneously.

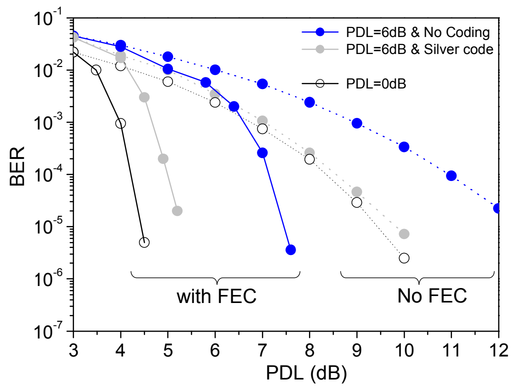

Fig. 6 BER performance of PT codes with FEC for PDL = 6dB. The solid (dotted) lines correspond to (no) FEC combination with PT coding.

V. EXPERIMENTAL RESULTS OF PDL MITIGATION

We have realized an experimental demonstration of an OFDM system with PT coding. The performances of Silver, Golden and Alamouti codes have been compared to the uncoded scheme. This work results from a collaboration with the Karlsruhe Institute of Technology [17].

The experimental arrangement is shown in Fig. 7. The two transmitted OFDM signals consist in 512 subcarriers,

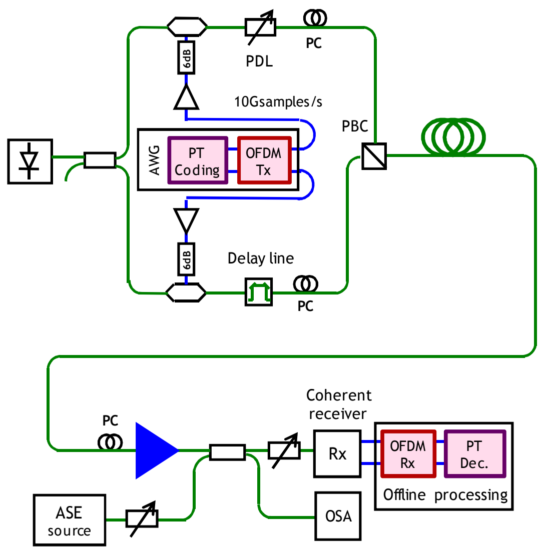

Fig. 7 PolMux OFDM experimental set-up.

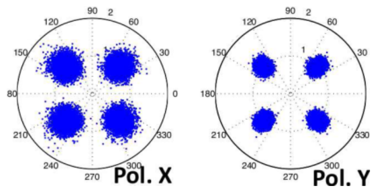

Fig. 8 Equalized uncoded subcarriers constellations for PDL=6dB.

where 480 carry data (16 on each side of the spectrum are set to zero to avoid severe sideband filtering effects). In order to compare the performance of different PT codes with the uncoded case, we assign alternatively QPSK uncoded, Silver-, Golden- and Alamouti-codes to the subcarriers. In order to work at the same effective transmission bit rate, Alamouti-coded subcarriers use a 16-QAM modulation format whereas the others use a QPSK format. After IFFT, we add a 10 samples length cyclic prefix to avoid ISI. Then, 12 training OFDM symbols are inserted every 500 data OFDM symbols for synchronization and channel estimation. Both OFDM baseband signals are generated using only one Tektronics 7122B arbitrary waveform generator (AWG). The sampling rate of the AWG outputs is 10Gsamples/s, corresponding to a total effective bit rate for the PolMux-OFDM system of 35.8 Gb/s.

The generation of the PT coding symbols requires the transmission of correlated symbols. Ideally, two AWGs would be required to generate the two complex symbols corresponding to each polarization. Having only one AWG, real OFDM signals have to be generated for each polarization and transmitted on the two outputs of the AWG. We can achieve this by letting the OFDM spectrum satisfy the Hermitian symmetry (i.e. half of the subcarriers are complex conjugate of the others) [18]. Note that all symbol subcarriers are considered as data for BER computation.

The OFDM signals are linearly transferred from electrical to optical domain using single-drive MZ modulators biased at null transmission point. The PDL is introduced at emission by attenuating one of the two polarizations. We call PDL, the attenuation value in dB. The optical path difference between the two polarizations is perfectly compensated by an optical delay line. The two optical OFDM signals are then multiplexed in polarization using a polarization beam combiner (PBC). A polarization controller (PC) is inserted to vary the state of polarization on the coherent receiver (there is no polarization alignment). The PolMux-OFDM signal is amplified and combined with ASE source. The OSNR level is measured using an Optical Spectrum Analyzer (OSA). The same laser (Δν~100KHz) is used for signal and local oscillator (i.e. self-homodyne detection). The coherent receiver consists of a polarization-diversity receiver followed by 4 balanced photodiodes to generate the electrical I and Q components for each polarization. Finally, these 4 electrical signals are sampled using a 40Gsamples/s real-time oscilloscope and ~10^6 symbols are recorded. The signal is finally down-sampled to 10Gsamples/s before off-line processing.

Off-line processing starts with the synchronization of received signal [19], then the cyclic prefix is removed from the OFDM blocks and the FFT is performed. Using the training sequence, the channel is estimated for each subcarrier as described in [20]. The phase recovery is realized by the Viterbi-Viterbi algorithm where the phase is estimated and averaged over the QPSK uncoded subcarriers. Finally ML decoding of the PT coded and uncoded subcarriers is performed. Fig. 8 shows the two received constellations for a PDL of 6dB in case of equalized uncoded QPSK subcarriers constellations for the 2 received polarizations.

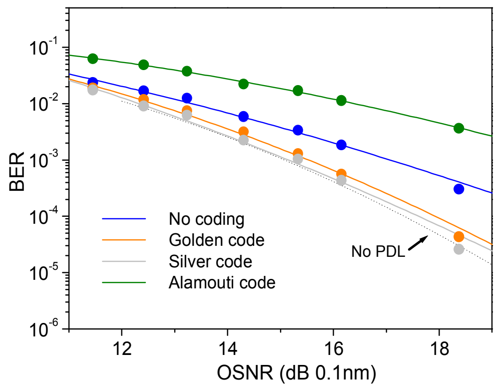

Fig. 9 BER comparison of PT-codes for PDL=3dB (lines are obtained by interpolation).

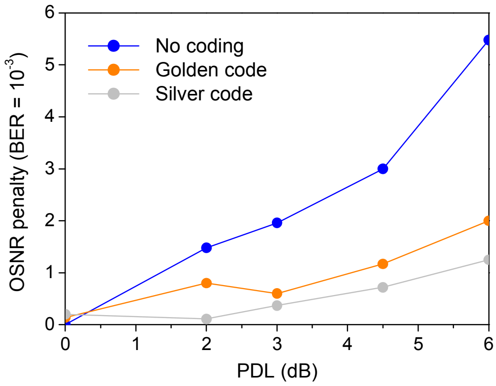

Fig. 10 ONSR penalty versus PDL with and without PT coding.

We evaluate the BER by averaging over the different subcarriers groups (uncoded, Silver, Golden and Alamouti). Without PDL, the uncoded, Silver and Golden schemes have exactly the same performance. Note that the Alamouti code performance presents a penalty of about 4dB due to the use of the 16QAM format. In Fig. 9, for a PDL of 3dB, there is a coding gain of about 1.5dB using PT codes compared to the uncoded scheme. Fig. 10 presents OSNR penalty at BER=10^{-3} induced by PDL. For a PDL of 6dB, the penalties are about 5.5dB without coding but only 1.2dB with the Silver code.

VI. CONCLUSION

We have introduced PT coding to mitigate PDL in PolMux OFDM fiber transmission systems. We have considered two full-rate codes initially designed for wireless communications: the Golden code and the Silver code. Both codes do not introduce any redundancy and have good performances. PT coding is useless against dispersive effects (PMD) but can efficiently mitigate the PDL impairments. Indeed, important coding gains have been observed. From the optical channel model, we have seen that performances are no longer based on the same criteria as in Rayleigh fading channel and that the Silver code outperforms the Golden code in this configuration. PT coding represents a very powerful solution for future generation of transmissions systems.

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