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A New Incomplete Decode-and-Forward Protocol

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***Abstract*— In this work, we explore the introduction of distributed space-time codes in decode-and-forward (DF) protocols. We propose a new Incomplete DF protocol, based on a partial decoding at the relays. This strategy allows the new protocol to bring both full diversity and full symbol rate. Outage probabilities and simulation results show that the Incomplete DF protocol has better performance than any existing DF protocol and than NAF protocols using the same space-time codes.**

I. Introduction

Diversity techniques have been developed in order to combat fading on wireless channels. Recently, a new diversity technique has been proposed with cooperative systems [1]. Different nodes in the network cooperate in order to form a MIMO system array and exploit space-time diversity. Cooperation protocols have been classified in three main families: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF).

DF protocols require more processing than AF ones, as the signals have to be decoded at relay before being forwarded. However, if signals are correctly decoded at relays, performance are better than those of AF protocols, as noise is deleted. In some context, DF protocols are expected to play an important role. In particular for multi-hop systems, it has been proven in [2] that using a DF strategy is necessary.

There are few DF protocols in litterature unlike AF protocols. Existing DF protocols usually do not succeed to bring both full diversity and full symbol rate. LTW[1] DF protocol in [3] has a full diversity order but a rate of $\frac{1}{2}$ symbol per channel use (symb. pcu), while NBK[1] DF protocol in [4] have a rate of 1 symb. pcu, but no diversity. The only proposed solution to this problem is the Dynamic DF (DDF) protocol [5] which succeeds to bring both full diversity and a rate of 1 symb. pcu. However its implementation is quite complex, and a feasible DDF protocol is not proposed.

In order to define a DF protocol with both full rate and full diversity, we think to introduce distributed ST codes in DF protocols, in the same way they have been successfully used in AF protocols, and in particular in the NAF protocol [6]. We propose here a new Incomplete DF protocol, based on partial decoding at the relays, which has full rate and full diversity. Outage probabilities calculations and simulation results have been conducted to validate this approach, and prove that Incomplete DF has better peformance than any existing DF protocol, and than the NAF protocols using the same ST codes.

[1]named by its authors initials

II. System model

We consider a wireless network with $N+1$ sources and one destination. As the channel is shared in a TDMA manner, each user is allocated a different time slot, and the system can be reduced to a relay channel with one source, N relays and one destination. The $N+1$ sources will play the role of the source in succession, while the others will be used as relays.

The channel links are Rayleigh, slow fading, so we can consider their coefficients as constant during the transmission of at least one frame. We focus here on the protocol, so for simplicity, we assume uniformly distributed energy.

We consider half-duplex terminals; they cannot receive and transmit at the same time. Moreover, all terminals are equipped with only one antenna; the MIMO case is not considered in this work.

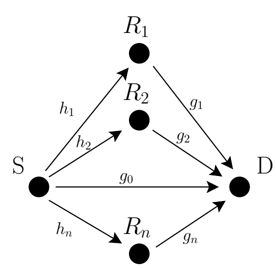

Fig. 1. System model : relay channel with one source, N relays and one destination

In the next sections, we will use the notation given on Figure 1. The channel coefficient of the link between source S and destination D is g_0, the one between source S and relay R$_i$ is h_i and the one between relay R$_i$ and destination D is g_i.

III. Incomplete DF protocol

In order to solve the problem of low rates or low diversity of existing DF protocols, we propose to introduce space-time codes in DF protocols, in the same way than in the NAF protocol, which leads to the definition of the new Incomplete DF. This protocol is designed to be used with a $2N \times 2N$ algebraic ST code, with N the number of relays. In the following, we will define the new protocol for the general case and give an example for the 2-relay case.

A. Transmission scheme

Let's consider the $2N \times 2N$ algebraic ST code C which can be either a TAST (Threaded Algebraic ST) code [7] or a perfect code [8]. Both families of codes have a codeword

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$$X = \begin{bmatrix} X_1 & X_2 & \dots & X_{2N-1} & X_{2N} \\ \gamma\sigma(X_{2N}) & \sigma(X_1) & \dots & \sigma(X_{2N-2}) & \sigma(X_{2N-1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma\sigma^{2N-2}(X_3) & \gamma\sigma^{2N-2}(X_4) & \dots & \sigma^{2N-2}(X_1) & \sigma^{2N-2}(X_2) \\ \gamma\sigma^{2N-1}(X_2) & \gamma\sigma^{2N-1}(X_3) & \dots & \gamma\sigma^{2N-1}(X_{2N}) & \sigma^{2N-1}(X_1) \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_{2N-1} \\ L_{2N} \end{bmatrix} \quad (1)$$

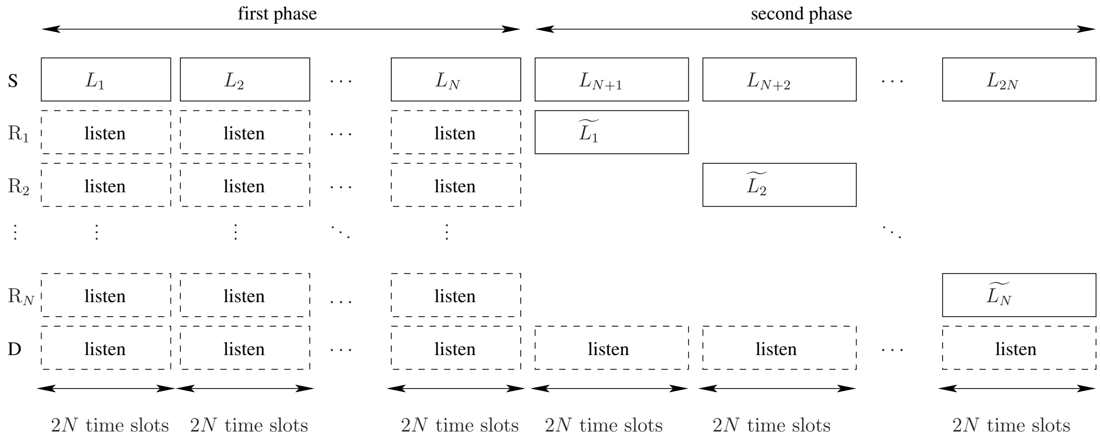

Fig. 2. Transmission frame of the Incomplete DF protocol

which can be written in the form (1) where the X_i, $i \in \{1,\dots,2N\}$, are elements of the ring of integers of K, a cyclic extension field of $Q(i)$ of dimension 2^{2N} (the X_i are linear combinations of $2N$ information symbols), σ is the generator of the Gallois group $K/Q(i)$ and γ is an element of either K or $Z(i)$ used to separate the layers of the codeword. Overall $4N^2$ information symbols are send in the codeword. Let's call L_k, $k \in \{1,\dots,2N\}$, the lines of the codeword matrix.

In order to implement this code in a N-relay channel, we define the transmission frame described in Figure 2. The transmission of one frame lasts $2N \times 2N = 4N^2$ channel uses. There are two main phases: during the first one, which lasts $2N \times N = 2N^2$ channel uses, the source sends the N first lines of the codeword matrix in succession and the N relays listen. During the second phase, which also lasts $2N \times N = 2N^2$ channel uses, the source sends the N last lines of the codeword, while the N relays send the decoded version of the N first lines. Relay R_i sends the decoded version of the i^{th} line of the codeword while source sends the $(N+i)^{\text{th}}$ line. The destination keeps listening during the whole transmission.

The rate is then $\frac{4N^2}{4N^2} = 1$ symbol pcu.

B. Partial decoding at the relays

The difficulty of the new transmission scheme is decoding at relays. Indeed, a full DF scheme would mean that relays have to decode the $4N^2$ symbols s_i of the original constellation from only $2N \times N = 2N^2$ received signals.

The idea of the Incomplete DF is to look at the received signals as elements X_k, $k \in \{1,\dots,2N\}$, of the ring of integers of the field K, and to decode them and their conjugates, without trying to decode the information symbols s_i, $i \in \{1,\dots,4N^2\}$. Indeed, knowledge of the s_i is not necessary at relays, as soon as they know the elements $X_k \in K$ that have to be forwarded. Partial decoding at relays is sufficient.

The partial decoding will be more detailed and explained in the sequel by considering an example.

C. Selection between the Incomplete DF and the SISO cases

DF protocols assume that signals are correctly decoded at the relay during the first phase of the transmission, which is obviously not always the case. That is why we have to guarantee the first phase of the transmission. In literature, a selection based on the source-relay links quality was made [9]. The used criterion is the outage probability. Indeed, according to Shannon theorem, if the link between source and relay R_i is in outage, no detection is possible at relay R_i without error. In the other case, detection is possible and we use a DF protocol assuming that no error occurs at relay R_i.

In our case, the outage probability of a source-relay R_i link is given by $P_O = P\left\{\log(1 + \mathsf{SNR}|h_i|^2) < 2R\right\}$ where R is the global spectral efficiency. The spectral efficiency of the source-relay link is twice since the same information is send in two times less channel uses.

In this work, we chose to use only the relays that can decode correctly the signals, and so for which the source-relay link is not in outage. That means that if $N_u \geq 1$ source-relay links are good, we use DF protocol with N_u relays, and if none of them is good, we use a non-cooperative strategy.

IV. Example of Incomplete DF implementation

Considering a 2-relay cooperative system, we propose to use a TAST code in a distributed manner, associated to the Incomplete DF protocol. We use here a 4×4 TAST code constructed using the cyclotomic field $K = Q(i, \theta)$, where $\theta = e^{i\frac{\pi}{8}}$, the generator of the Gallois group $\sigma : \theta \longmapsto i\theta$ and $\phi = e^{i\frac{\pi}{32}}$. The codeword is

$$X = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ \phi\sigma(X_4) & \sigma(X_1) & \sigma(X_2) & \sigma(X_3) \\ \phi\sigma^2(X_3) & \phi\sigma^2(X_4) & \sigma^2(X_1) & \sigma^2(X_2) \\ \phi\sigma^3(X_2) & \phi\sigma^3(X_3) & \phi\sigma^3(X_4) & \sigma^3(X_1) \end{bmatrix}$$

where

$$\begin{aligned} X_1 &= s_1 + \theta s_2 + \theta^2 s_3 + \theta^3 s_4 \\ X_2 &= s_5 + \theta s_6 + \theta^2 s_7 + \theta^3 s_8 \\ X_3 &= s_9 + \theta s_{10} + \theta^2 s_{11} + \theta^3 s_{12} \\ X_4 &= s_{13} + \theta s_{14} + \theta^2 s_{15} + \theta^3 s_{16} \end{aligned}$$

The codeword is sent in the way described in Figure 2.

Elements X_1, X_2, X_3 and X_4 of the ring of integers of the cyclotomic field $Q(i, e^{i\frac{\pi}{8}})$ and their conjugates have to be recovered from the signals $y_1^{r_j}$ to $y_8^{r_j}$ received at the relay R_j, $j \in \{1, 2\}$. We propose here 2 different methods for the partial decoding.

A. Exhaustive decoding (dimension 4)

We will decode the X_k, $k \in \{1, \ldots, 4\}$ and their conjugates $\sigma(X_k)$ at relays by an exhaustive search. The difficulty is that we send X_k and $\sigma(X_k)$ which cannot be decoded separately as they are conjugates.

Let's assume the s_i, $i \in \{1, \ldots, 16\}$, belong to a constellation C. We can define a new constellation C_1 to which the X_k belong, and a corresponding constellation C_2 to which their conjugates $\sigma(X_k)$ belong.

Decoded versions of the X_k and their conjugates $\sigma(X_k)$ are obtained by minimizing the distance between $Z \in C_1$ and the received signal corresponding to X_k and the distance between $\sigma(Z) \in C_2$ and the received signal corresponding to $\sigma(X_k)$. We decide to minimize the sum of these two distances so that

$$\{\widetilde{X_1}, \widetilde{\sigma(X_1)}\} = \arg \min_{Z, \sigma(Z)} \left\{ \left| \frac{y_1^{r_j}}{h_1} - Z \right|^2 + \left| \frac{y_8^{r_j}}{h_1} - \sigma(Z) \right|^2 \right\}$$

The same way, we can obtain $\{\widetilde{X_2}, \widetilde{\sigma(X_2)}\}$, $\{\widetilde{X_3}, \widetilde{\sigma(X_3)}\}$ and $\{\widetilde{X_4}, \widetilde{\sigma(X_4)}\}$.

However, this exhaustive decoding can be quite complex if a high constellation size is considered. Indeed, if the information symbols s_i belong to a q-QAM constellation, then, the X_k have to be decoded in a new constellation of q^4 elements.

B. Two steps exhaustive decoding (dimension 2)

A slight modification can reduce this high complexity.

We can notice that X_1 and its second conjugate $\sigma^2(X_1)$ can be rewritten in the form

$$\begin{bmatrix} X_1 \\ \sigma^2(X_1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \theta \\ 1 & -\theta \end{bmatrix}}_{M} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \tag{2}$$

where $R_1 = (s_1 + \theta^2 s_3)$ and $R_2 = (s_2 + \theta^2 s_4)$ are elements of the field $Q(e^{i\frac{\pi}{4}})$ of dimension 2 over $Q(i)$. As $\frac{1}{\sqrt{2}}M$ is a rotation matrix, by a simple multiplication by the Hermitian of M, we can obtain R_1 and R_2 from X_1 and $\sigma^2(X_1)$.

In order to take advantage of this particularity, the idea is that the source sends the first and third lines of the codeword matrix during the first phase of the transmission and the second and fourth lines during the second phase of the transmission. The partial decoding at relays is then done in two steps. First we compute the matrix product

$$\begin{bmatrix} R_1' \\ R_2' \end{bmatrix} = \frac{1}{2} M^H \begin{bmatrix} \frac{y_1^{r_k}}{h_1} \\ \frac{y_8^{r_k}}{h_1} \end{bmatrix}$$

Then we decode elements R_1 and R_2 of the field $Q(e^{i\frac{\pi}{4}})$ in an exhaustive way. Finally X_1 and its conjugate $\sigma^2(X_1)$ can be easily deduced from equation (2).

This second method allows to decrease complexity a lot. Indeed, the exhaustive search is now used in a constellation of q^2 elements instead of q^4, which is quite reasonable.

It is to be noted that this decoding method cannot be applied to 4×4 perfect codes, that is why we use TAST code.

V. Performance

A. Outage probability

Outage probability is given by the formula $P_{\text{out}} = P\left\{\log\det(I + \mathsf{SNR}HH^H) < R\right\}$ where SNR is the signal to noise ratio, H is the (equivalent) channel matrix of the considered system and R is the spectral efficiency.

We have to distinguish two cases: when we use cooperation or not. When $N_u \geq 1$ source-relay links are not in outage, we use the Incomplete DF cooperation scheme with N_u relays. Assuming that the N_u usable relays are the N_u first ones, we can write

$$\begin{aligned} P_{\text{out},N_u} &= P\Big\{ \tfrac{1}{2N_u} \log\Big(\textstyle\prod_{i=1}^{N_u} \left(1 + \tfrac{\mathsf{SNR}}{2}\left(3|g_0|^2 + g_i|^2\right)\right. \\ &\qquad\qquad \left. + \tfrac{\mathsf{SNR}^2}{2}|g_0|^4\right)\Big) < R \Big\} \\ &= P\Big\{ \log \textstyle\prod_{i=1}^{N_u} \Big(1 + \tfrac{\mathsf{SNR}}{2}\left(3|g_0|^2 + g_i|^2\right) \\ &\qquad\qquad + \tfrac{\mathsf{SNR}^2}{2}|g_0|^4\Big) < 2N_u R \Big\} \end{aligned} \tag{3}$$

The probability of this case happening, which means the probability of having the other $N - N_u$ relays in outage, is

$$\begin{aligned} P_{O,N-N_u} &= \textstyle\prod_{i=1}^{N_u} P\left\{\log\left(1 + \mathsf{SNR}|h_i|^2\right) > 2R\right\} \\ &\quad \textstyle\prod_{i=N_u+1}^{N} P\left\{\log\left(1 + \mathsf{SNR}|h_i|^2\right) < 2R\right\} \end{aligned} \tag{4}$$

When all source-relay links are in outage, we use the non-cooperative scheme

$$P_{\text{out},0} = P\left\{\log\left(1 + \mathsf{SNR}|g_0|^2\right) < R\right\} \tag{5}$$

which happens with a probability

$$P_{O,N} = \prod_{i=1}^{N} P\left\{\log\left(1 + \mathsf{SNR}|h_i|^2\right) < 2R\right\} \tag{6}$$

Finally we can write in the general case

$$P_{\text{out}} = \sum_{N_u=0}^{N} C_{N_u}^{N} P_{\text{out},N_u} P_{O,N-N_u} \tag{7}$$

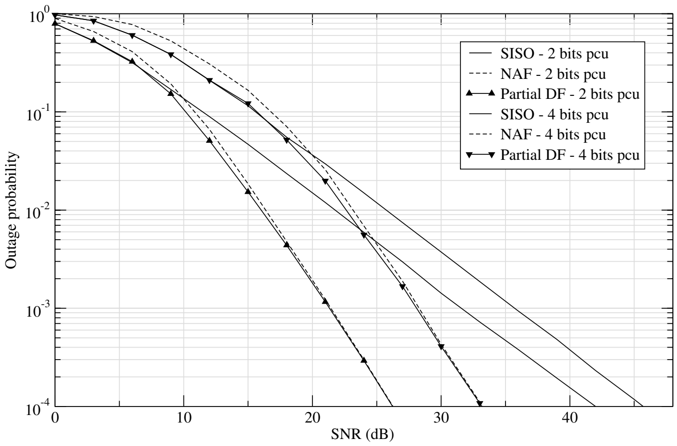

Fig. 3. Outage probabilities for 1 relay chosen out of 3

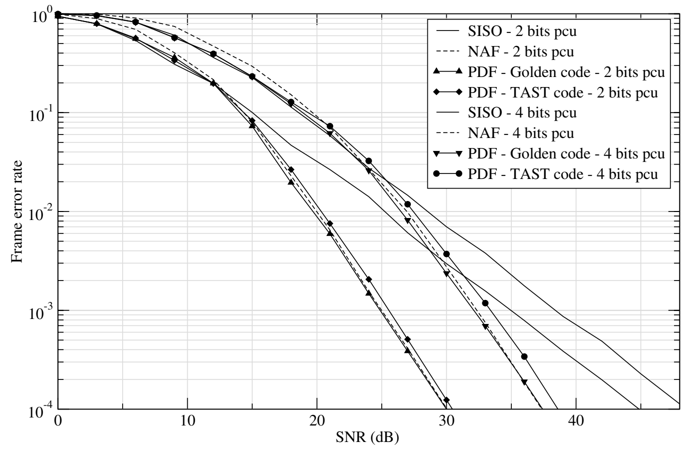

Fig. 5. Frame error rates for 1 relay chosen out of 3

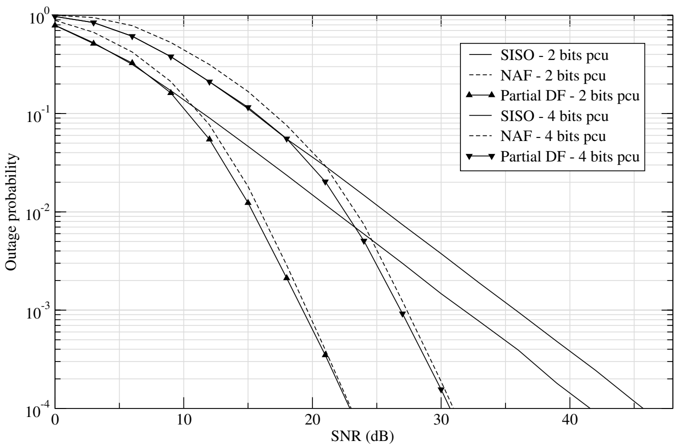

Fig. 4. Outage probabilities for 2 relays chosen out of 4

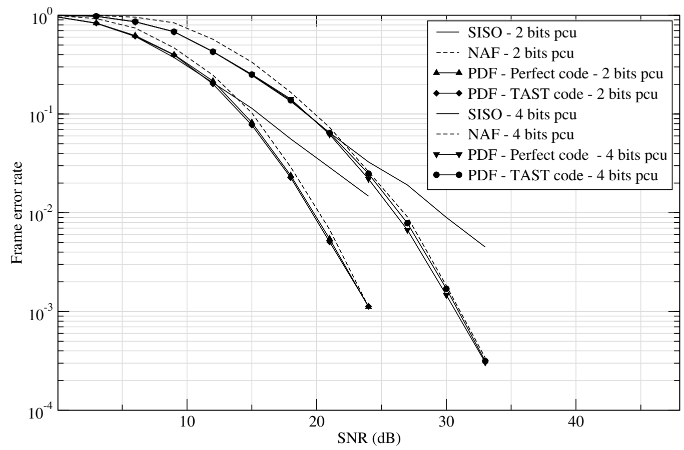

Fig. 6. Frame error rates for 2 relays chosen out of 4

Figures 3 and 4 represent the outage probabilities of the SISO, NAF and new DF protocols as functions of the SNR at spectral efficiencies of 2 and 4 bits per channel use. In order to enhance the performance, we add a relay selection for all cooperative schemes. In the one-relay case, the relay is chosen as the best of 3 reachable relays, and in the two-relay case, relays are the two best ones between four relays. In practice, in a first step the two best relays are selected, and in a second step, the DF protocol determine which of these two relays can be used (i.e. source-relay link not in outage) and so which strategy is to be chosen between SISO, cooperative with 1 relay or cooperative with 2 relays. The AF protocol always use both relays. The new DF protocol brings a slight dB gain over the NAF protocol. Moreover, and more interesting is the fact that it has good performance at low SNR.

B. Simulation results

Simulations have been run for one relay with both the Golden code and a 2×2 TAST code, and for two relays with both a 4×4 perfect code and a 4×4 TAST code for spectral efficiencies of 2 and 4 bits per channel use. The same relay selection as in the previous subsection has been applied.

Figures 5 and 6 represent the frame error rates of the SISO, NAF and new DF protocols as functions of the SNR. The good performance for low and high SNR proved by the outage probability study is confirmed by simulation results. In the one-relay case, we obtain asymptotic gains of 0.7 and 1.2 dB for spectral efficiencies of 2 and 4 bits pcu respectively. Moreover, we can see (especially for 4 bits pcu) that the proposed DF protocol has better performance at low SNR. Same remarks can be done in the two-relay case. The proposed DF protocol and the NAF protocol have nearly the same performance (nearly zero asymptotic gain), but the proposed DF protocol outperforms the NAF at low SNR.

As perfect codes have a non-vanishing-determinant, on the contrary of TAST codes, they give slightly better performance. However, when we use 2 or more relays, the partial decoding of the information at relays induces more complexity, as the two-step decoding method described in subsection IV cannot be used.

C. Diversity-Multiplexing gain Tradeoff (DMT)

The DMT has been introduced in [10] in order to evaluate the asymptotic performance of ST codes. A diversity gain $d(r)$ at a multiplexing gain r is given by

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log P_{\mathrm{out}}(r \log \mathsf{SNR})}{\log \mathsf{SNR}} = -d(r)$$

Let's define $u_0 \doteq -\frac{\log|g_0|^2}{\log \mathsf{SNR}}$ so we can note $|g_0|^2 \doteq \mathsf{SNR}^{-u_0}$ where the notation $\doteq$ denotes an asymptotic behavior when $\mathsf{SNR} \to \infty$. In the same way, we define u_i and v_i such as $|g_i|^2 \doteq \mathsf{SNR}^{-u_i}$ and $|h_i|^2 \doteq \mathsf{SNR}^{-v_i}$.

The outage probability of this new Incomplete DF scheme is given in equations (3) to (7) in subsection V-A. In order to compute the DMT of this cooperative strategy, we have to study the asymptotic behavior of these equations when the SNR grows to infinity.

In the case of signals being correctly decoded at N_u relays, equation (3) asymptotically becomes

$$\begin{aligned} P_{\text{out},N_u} &\doteq P\Big\{ \textstyle\sum_{i=1}^{N_u} \log(\mathsf{SNR}^{1-v_0} + \mathsf{SNR}^{1-v_i} \\ &\qquad + \mathsf{SNR}^{2-2v_0}) < 2N_u r \log \mathsf{SNR}\Big\} \\ &\doteq P\left\{ \textstyle\sum_{i=1}^{N_u} \max(1-v_i, 2-2v_0) < 2N_u r \right\} \\ &\doteq \mathsf{SNR}^{-d_{out,N_u}(r)} \end{aligned}$$

As $\sum_{i=1}^{N_u}(1-v_i) < 2N_u r$ gives $N_u(1-2r) < \sum_{i=1}^{N_u} v_i$ and $\sum_{i=1}^{N_u}(2-2v_0) < 2N_u r$ gives $1-r < v_0$, we obtain the diversity-multiplexing gain tradeoff

$$d_{out,N_u}(r) = \inf\left(v_0 + \sum_{i=1}^{N_u} v_i\right) = (1-r) + N_u(1-2r)^+ \quad (8)$$

This case occurs when $N - N_u$ of the source-relay links are in outage and the others are not (equation (4)).

$$\begin{aligned} P_{O,N-N_u} &\doteq \textstyle\prod_{i=1}^{N_u} P\left\{\log \mathsf{SNR}^{1-u_i} > 2r \log \mathsf{SNR}\right\} \\ &\qquad \textstyle\prod_{i=N_u+1}^{N} P\left\{\log \mathsf{SNR}^{1-u_i} < 2r \log \mathsf{SNR}\right\} \\ &\doteq \textstyle\prod_{i=1}^{N_u} 1 \prod_{i=N_u+1}^{N} P\{1-u_i < 2r\} \\ &\doteq \textstyle\prod_{i=N_u+1}^{N} \mathsf{SNR}^{-(1-2r)} \end{aligned}$$

so the diversity-multiplexing gain tradeoff is

$$d_{O,N-N_u}(r) = (N-N_u)(1-2r)^+ \quad (9)$$

In the case of all source-relay links being in outage, equation (5) asymptotically becomes

$$P_{\text{out},0} \doteq P\left\{\log \mathsf{SNR}^{1-v_0} < r \log \mathsf{SNR}\right\} \doteq P\{1-v_0 < r\}$$

with the diversity-multiplexing gain tradeoff

$$d_{out,0}(r) = 1 - r \quad (10)$$

This case occurs when all source-relay links are in outage (equation (6))

$$\begin{aligned} P_{O,N} &\doteq \textstyle\prod_{i=1}^{N} P\left\{\log \mathsf{SNR}^{1-u_i} < 2r \log \mathsf{SNR}\right\} \\ &\doteq \textstyle\prod_{i=1}^{N} P\{1-u_i < 2r\} \\ &\doteq \textstyle\prod_{i=1}^{N} \mathsf{SNR}^{-(1-2r)} \end{aligned}$$

so the diversity-multiplexing gain tradeoff is

$$d_{O,N}(r) = N(1-2r)^+ \quad (11)$$

Finally we can write

$$P_{\text{out}} \doteq \sum_{N_u=0}^{N} C_{N_u}^{N} \mathsf{SNR}^{-d_{out,N_u}(r)} \mathsf{SNR}^{-d_{O,N-N_u}(r)}$$

and the total diversity-multiplexing gain tradeoff is

$$\begin{aligned} d(r) &= \max_{N_u \in \{0,\dots,N\}} (d_{out,N_u}(r) + d_{O,N-N_u}(r)) \\ &= (1-r) + N(1-2r)^+ \end{aligned} \quad (12)$$

One can remark that this is exactely the same DMT as the one of the NAF protocol, outperforming the ones of the LTW and NBK DF protocols. The DMT of the DDF protocol is still better, but Incomplete DF implementation is much easier, and practical ST codes are known.

VI. Conclusion

We propose a DF protocol using distributed ST codes that provides both full diversity and full rate, as the best known AF protocols, and unlike the existing LTW and NBK DF protocols. This new protocol is based on a partial detection of the signal at the relays. The received signals at relays are decoded as elements of the ring of integers of the considered number field without decoding the information symbols. Two partial decoding methods are proposed: the exhaustive search and an method based on the decomposition of the decoding in two steps based on the code structure. This last method allows a considerable reduce of complexity.

The diversity-multiplexing gain tradeoff is proved to be the same as the one of the NAF protocol which is the best known AF protocol. Besides outage probability and simulation results prove that this new protocol gives slightly better performance than the NAF protocol.

DF protocols can be very important in a non-line-of-sight or multihop context. Applications of the new protocol to these systems will be investigated in future works.

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