Diophantine Approximation Approach for Incomplete Decoding at Relays

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***Abstract*— In this paper, we propose a low-complexity method to perform the incomplete decoding needed at relays for the Incomplete Decode-and-Forward. The decoding problem is considered as a diophantine approximation search. We propose two modifications of Cassels' algorithm to approximate an element $x \in \mathbb{Q}(i, \theta)$ by a linear combination $a + \theta b$ of symbols taken in a Z-PAM. Simulation results show that the use of diophantine approximation induces only a small loss in performance, while reducing significantly the processing complexity.**

I. Introduction

Cooperation has been developed in order to exploit space-time diversity even when small terminals are not equipped with several antennas: different nodes in the network cooperate in order to form a MIMO system array [1], [2]. Cooperation protocols have been classified in three main families: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF).

DF protocols require more processing than AF ones, as the signals have to be decoded at relay before being forwarded. However, if signals are correctly decoded at relays, performance are better than those of AF protocols, as noise is deleted. Moreover, DF protocols can be more performant in some scenarios. For example, it has been proven in [3] that in a multihop context it is necessary to use a DF protocol at some relays to regenerate the signals. A full AF strategy would add more noise at each hop and make signals no longer decodable.

In [4], we proposed the Incomplete DF protocol providing both full rate of 1 symbol per channel use and full diversity order (K+1) (where K is the number of relays). The main idea of this protocol is to perform only an incomplete decoding at the relays. Based on the structure of the distributed TAST code, a two-step decoding method was defined, allowing to reformulate the problem as a search in two-dimension, whatever the number of relays. In [4] the incomplete decoding is performed using an exhaustive search. However, this exhaustive search induces a high processing complexity.

In [5], authors proposed to use a diophantine approximation to decode the Golden code in a 2×1 MISO channel. The difficulty is then the rank deficiency of the system. In the Incomplete DF protocol, decoding at relay also suffers from a rank deficiency. So, based on the same idea, we propose to approximate coded symbols at relay using diophantine approximation algorithms. We present an adaptation of the well-known Cassels' algorithm for approximating an element $x \in \mathbb{Q}(i, \theta)$ with $\theta \in \mathbb{R}$ by a linear combination $a + \theta b$, where a and b are symbols taken in a Z-PAM. In order to combine diophantine approximation with the two-step decoding proposed in [4], we also propose an other modification of this algorithm to the case where $\theta = e^{i\frac{\pi}{4}}$. Simulation results show that the use of a diophantine approximation at relay induces only a small loss in performance, highly counterbalanced by the decrease of processing complexity.

II. Preliminaries

A. Channel model

We consider a wireless network with $N + 1$ sources and one destination. As the channel is shared in a TDMA manner, each user is allocated a different time slot, and the system can be reduced to a relay channel with one source, N relays and one destination. The $N + 1$ sources will play the role of the source in succession, while the others will be used as relays.

The channel links are assumed to be Rayleigh distributed and slow fading, so their coefficients can be considered as constants during the transmission of at least one frame. Besides, we suppose a symmetric scenario, i.e. all the channel links are subject to the same average signal-to-noise ratio (SNR). A uniform energy distribution is assumed.

Considered terminals are half-duplex; they cannot receive and transmit at the same time. They are equipped with only one antenna; the MIMO case is not considered in this work.

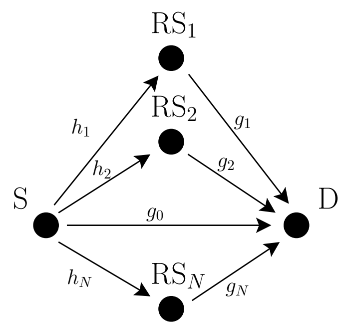

Fig. 1. System model : relay channel with one source, N relays and one destination

In the next sections, notation given on Figure 1 will be used. The channel coefficient of the link between source S and destination D is g_0, the one between source S and relay

RS_n, $n \in \{1, \dots, N\}$, is h_n and the one between relay RS_n and destination D is g_n.

There is no channel state information (CSI) at the source, the destination is supposed to know all the channel coefficients g_n, which is necessary for the decoding of the information, and each relay RS_n is assumed to know its corresponding source-relay channel coefficient h_n.

B. Incomplete DF protocol

The IDF protocol has been proposed in [4]. Its transmission frame is very similar to the one of the NAF protocol [6]. During a first phase, the source broadcasts N symbols that are received by all the N relays. During the second phase, the source keeps transmitting and each relay $n \in \{1, ..., N\}$ decodes and forwards the n^{th} symbol received in the first phase.

This strategy allows to transmit at a rate of 1 symbol per channel use (pcu) and achieves full diversity $N+1$. Its diversity-multiplexing gain tradeoff (DMT) is the same as the one achieved by the NAF protocol:

$$d^*(r) = (1-r)^+ + N(1-2r)^+ \tag{1}$$

In order to reach this theoretic DMT, optimal space-time codes such as perfect [7], [8] have to be used in a distributed manner. Coded symbols are then elements of the ring of integers $\mathcal{O}_{\mathbb{K}}$ of $\mathbb{K}$, a cyclic extension field of $\mathbb{Q}(i)$ of dimension 2^{2N}, i.e. they are linear combinations of $2N$ information symbols each. Thus the challenge of this protocol lies in the decoding at relays. Indeed, coded symbols containing a total of $4N^2$ information symbols have to be decoded from only $2N^2$ received signals.

The main idea of the Incomplete DF protocol is to estimate received signals as elements $x_k \in \mathcal{O}_{\mathbb{K}}$, $k \in \{1, \dots, 2N\}$, without having to decode the information symbols s_j, $j \in \{1, \dots, 4N^2\}$. Indeed, the knowledge of the s_j is not necessary at relays, as soon as they know the signals x_k that have to be forwarded.

Decoding at destination can be performed by using ML lattice decoders such as a Schnorr-Euchner [9] or a sphere decoder [10].

C. Definition of a diophantine approximation

There exist two types of diophantine approximation.

Definition 1: A homogeneous diophantine approximation of $\zeta \in \mathbb{R}$ is a fraction $\frac{p}{q} \in \mathbb{Q}$ such that $|\zeta - \frac{p}{q}|$ or $D(p,q) = |q\zeta - p|$ is small.

Definition 2: An inhomogeneous diophantine approximation of $\zeta \in \mathbb{R}$, given $\beta \in \mathbb{R}$, is a fraction $\frac{p}{q} \in \mathbb{Q}$ such that $D(p,q) = |q\zeta - p - \beta|$ is small.

Definition 3: A pair $(p,q) \in \mathbb{N}^2$ is a best (homogeneous or inhomogeneous) diophantine approximation if $\forall (p', q') \neq (p,q) \in \mathbb{N}^2$, we have:

$$q' \leq q \Rightarrow D(p', q') \geq D(p,q).$$

Cassels' algorithm has been proposed in [11] and explained in details in [12]. Given $\zeta, \beta \in \mathbb{R}$, this algorithm enumerates all best inhomogeneous approximations.

III. One-relay case

In this section, we suppose that the source is helped by only one relay RS. The Incomplete DF protocol can then be implemented with a 2×2 distributed STBC. We propose to use the distributed Golden code.

The Golden code is an algebraic code designed for a 2×2 MIMO system in [13] based on the cyclic division algebra of dimension 2, $\mathcal{A} = (\mathbb{Q}(i,\theta)/\mathbb{Q}(i), \sigma, \gamma)$, where $\theta = \frac{1+\sqrt{5}}{2}$ is the Golden number, $\sigma : \frac{1+\sqrt{5}}{2} \longmapsto \frac{1-\sqrt{5}}{2}$ and $\gamma = i$.

A codeword is given by

$$\mathbf{X} = \begin{bmatrix} \alpha(s_1 + \theta s_2) & \alpha(s_3 + \theta s_4) \\ i\sigma(\alpha)(s_3 + \sigma(\theta)s_4) & \sigma(\alpha)(s_1 + \sigma(\theta)s_2) \end{bmatrix}$$

where the s_j, $j \in \{1, \dots, 4\}$ are the information symbols taken in a QAM constellation and $\alpha = 1 + i - i\theta$. The elements of the code matrix are in $\mathcal{O}_{\mathbb{K}}$ the ring of integers of the number field $\mathbb{K} = \mathbb{Q}(i,\theta)$. Let's note them $x_1 = s_1 + \theta s_2$ and $x_2 = s_3 + \theta s_4$. The codeword is then:

$$\mathbf{X} = \begin{bmatrix} \alpha x_1 & \alpha x_2 \\ i\sigma(\alpha)\sigma(x_2) & \sigma(\alpha)\sigma(x_1) \end{bmatrix}. \tag{2}$$

The transmission frame is represented in Figure 2.

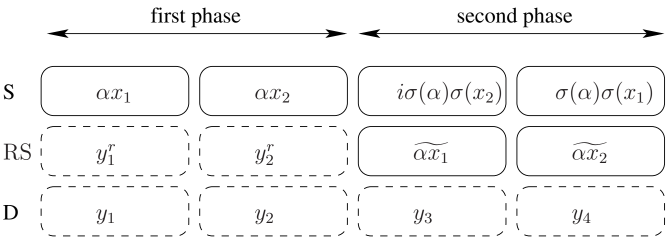

Fig. 2. Transmission frame of Incomplete DF for one relay implemented with a distributed Golden code. Sent signals are represented in continuous boxes and received signals in dashed boxes.

Elements x_1 and x_2 both contain two information symbols. They have to be recovered respectively from the received signals y_1^r and y_2^r.

In [4], an exhaustive decoding is performed at relays. This strategy induces a high processing complexity. In this paper, we propose to reduce this decoding complexity by using a diophantine approximation of the x_k, $k \in \{1, 2\}$.

Diophantine approximation only deals with real numbers. As in this case $\theta \in \mathbb{R}$, diophantine approximation perfectly fits to the decoding of the elements of the Golden codeword. The problem only has to be divided into its real and imaginary parts. Let's note

$$\widetilde{y_1^r} = \frac{y_1^r}{\sqrt{\rho}h_1\alpha} \qquad \text{and} \qquad \widetilde{y_2^r} = \frac{y_2^r}{\sqrt{\rho}h_2\alpha}.$$

Given $(\theta, Re(\widetilde{y_1^r})) \in \mathbb{R}^2$, we want to find $(Re(s_1), Re(s_2)) \in \sqrt{M}$-PAM such that

$$d = |Re(\widetilde{y_1^r}) - Re(s_1) - \theta Re(s_2)| \tag{3}$$

is minimized. Using the notation of definition 2, this problem can be identified with an inhomogeneous diophantine approximation where $\zeta \leftrightarrow -\theta$, $p \leftrightarrow Re(s_1)$, $q \leftrightarrow Re(s_2)$ and $\beta \leftrightarrow -Re(\widetilde{y_1^r})$. However, $Re(s_1)$ and $Re(s_2)$ have to be decoded in a $\sqrt{M}$-PAM. Cassels' algorithm thus has to be modified so as to provide (p, q) in a finite set. A change of basis provides (p, q) in a Z-PAM.

Input: y, θ, Z
Output: $\widehat{X}$

1 $\beta = -(y + (Z+1)(1+\theta))/2$;
2 $\alpha = -\theta$;
3 $\eta_0 = \alpha; \eta_1 = -1; \zeta_1 = -\beta$;
4 $p_0 = 0; p_1 = 1; P_1 = 0$;
5 $q_0 = 1; q_1 = 0; Q_1 = 0$;
6 $n = 2$;
7 **while** $\eta_{n-1} \neq 0 \wedge \zeta_{n-1} \neq 0 \wedge Q_{n-1} \leq Z$ **do**
8 $\quad a_n = \lfloor -\frac{\eta_{n-2}}{\eta_{n-1}} \rfloor$;
9 $\quad p_n = p_{n-2} + a_n p_{n-1}$;
10 $\quad q_n = q_{n-2} + a_n q_{n-1}$;
11 $\quad \eta_n = \eta_{n-2} + a_n \eta_{n-1}$;
12 $\quad$ **if** $Q_{n-1} \leq q_{n-1}$ **then**
13 $\quad\quad b_n = \lfloor -\frac{\zeta_{n-1} - \eta_{n-2}}{\eta_{n-1}} \rfloor$;
14 $\quad\quad P_n = P_{n-1} + p_{n-2} + b_n p_{n-1}$;
15 $\quad\quad Q_n = Q_{n-1} + q_{n-2} + b_n q_{n-1}$;
16 $\quad\quad \zeta_n = \zeta_{n-1} + \eta_{n-2} + b_n \eta_{n-1}$;
17 $\quad$ **else**
18 $\quad\quad P_n = P_{n-1} - p_{n-1}$;
19 $\quad\quad Q_n = Q_{n-1} - q_{n-1}$;
20 $\quad\quad \zeta_n = \zeta_{n-1} - \eta_{n-1}$;
21 $\quad$ **end**
22 $\quad n = n + 1$;
23 **end**
24 $P = 2P_n - (Z+1)$;
25 $Q = 2Q_n - (Z+1)$;
26 $\widehat{X} = P + \theta Q$;

Algorithm 1: Modified Cassels' algorithm for decoding symbols in a Z-PAM, with $\theta \in \mathbb{R}$

The modified Cassels' algorithm is given in Algorithm 1. The constraint on Q_{n-1} on line 7 allows to restrict the search to a finite set $\{1, \ldots, Z\}$. When $Q_{n-1} > Z$, the computation is stopped. The change of basis is done on line 1 and the reverse on lines 24 and 25.

The same processing is done to decode the imaginary part of the first coded symbol, as well as the real and imaginary parts of the second coded symbol.

The incomplete decoding using a diophantine approximation is not optimal. However, it allows a considerable drop of the decoding complexity at the relay side. Indeed, exhaustive decoding has a complexity order M^2 that can be easily reduced to M by separating the real and imaginary parts of the signal. When running the modified Cassels' algorithm for different size of constellation, we remark that it requires an average of 3 iterations for a BPSK, 4 iterations for a 4-PAM and 8 iterations for a 16-PAM. Based on these results, we conjecture that the average number of iterations of the algorithm is $2\sqrt{Z}$ for a Z-PAM. The processing complexity of the diophantine approximation is thus of the order $\sqrt{Z} = \sqrt[4]{M}$.

IV. Two-relay case

In this section, we suppose that the source is helped with two relays RS_1 and RS_2. The Incomplete DF protocol has to be implemented with a 4×4 distributed STBC. We propose to use a distributed TAST code. Such codes have a vanishing determinant and so do not achieve the DMT. However, we will show in the following that this drawback is counterbalanced by their code structure which allows a lower complexity decoding.

TAST codes, introduced in [14], are layered space-time codes. In this paper, we use the TAST code constructing using the cyclotomic field $\mathbb{K} = \mathbb{Q}(i, \theta)$, where $\theta = e^{i\frac{\pi}{8}}$, the generator of the Gallois group $\sigma : \theta \longmapsto i\theta$ and $\phi = e^{i\frac{\pi}{8}}$. The codeword is

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ \phi\sigma(x_4) & \sigma(x_1) & \sigma(x_2) & \sigma(x_3) \\ \phi\sigma^2(x_3) & \phi\sigma^2(x_4) & \sigma^2(x_1) & \sigma^2(x_2) \\ \phi\sigma^3(x_2) & \phi\sigma^3(x_3) & \phi\sigma^3(x_4) & \sigma^3(x_1) \end{bmatrix},$$

where, $\forall k \in \{1, \ldots, 4\}$, $x_k = s_{4k-3} + \theta s_{4k-2} + \theta^2 s_{4k-1} + \theta^3 s_{4k}$.

Elements x_1, x_2, x_3 and $x_4 \in \mathcal{O}_{\mathbb{K}}$ and their conjugates have to be recovered from the signals $y_1^{r_j}$ to $y_8^{r_j}$ received at the relay RS_j, $j \in \{1, 2\}$.

A. Two-step decoding

A method based on the structure of the TAST code is proposed in [4] to reduce the decoding complexity at relays. A slight modification can reduce the exhaustive decoding in a constellation of M^4 elements to two exhaustive decodings in a constellation of only M^2 elements.

We can notice that x_1 and its second conjugate $\sigma^2(x_1)$ can be rewritten in the form:

$$x_1 = (s_1 + \theta^2 s_3) + \theta(s_2 + \theta^2 s_4)$$
$$\sigma^2(x_1) = (s_1 + \theta^2 s_3) - \theta(s_2 + \theta^2 s_4)$$
$$\begin{bmatrix} x_1 \\ \sigma^2(x_1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \theta \\ 1 & -\theta \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \qquad (4)$$

where $z_1 = (s_1 + \theta^2 s_3)$ and $z_2 = (s_2 + \theta^2 s_4)$ are elements of the ring of integers of the field $\mathbb{Q}(e^{i\frac{\pi}{4}})$ of dimension 2 over $\mathbb{Q}(i)$. As $\frac{1}{\sqrt{2}}\mathbf{M}$ is a rotation matrix, a simple multiplication by $\mathbf{M}^\dagger$ allows to obtain z_1 and z_2 from x_1 and $\sigma^2(x_1)$.

In order to take advantage of this property, the idea is that the source sends the first and third lines of the codeword matrix during the first phase of the transmission and the second and fourth lines during the second phase of the transmission.

The incomplete decoding at relays is then done in two steps. First we compute the matrix product

$$\begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \frac{1}{2}\mathbf{M}^\dagger \begin{bmatrix} \frac{y_1^{r_k}}{\sqrt{\rho}h_1} \\ \frac{y_6^{r_k}}{\sqrt{\rho}h_1} \end{bmatrix}.$$

Then we decode elements z_1 and z_2 of the ring of integers of $\mathbb{Q}(e^{i\frac{\pi}{4}})$ in an exhaustive way as in [4]. Finally x_1 and its conjugate $\sigma^2(x_1)$ can be easily deduced from (4).

This trick allows to decrease considerably the diversity. Indeed, the exhaustive search is now performed in a constellation of M^2 elements instead of M^4.

B. Diophantine approximation with $\theta = e^{i\frac{\pi}{4}}$

In order to reduce the complexity even more, we want to combine the advantages of the two-step decoding method and the diophantine approximation. The problem is that θ is not a real number anymore.

However, we can show that if $\theta = e^{i\frac{\pi}{4}}$ the decomposition in real and imaginary part is more complex, but the diophantine approximation still can be used with a slight modification of the given algorithm. Thus the diophantine approximation can also be applied when using a distributed TAST code.

A coded symbol is in the form

$$s_c = s_1 + \theta s_2$$

where s_1 and s_2 are complex information symbols.

$$\begin{aligned} s_c &= \Re(s_1) + i\Im(s_1) + \frac{1}{\sqrt{2}}(1+i)\left(\Re(s_2) + i\Im(s_2)\right) \\ &= \Re(s_1) + \frac{1}{\sqrt{2}}(\Re(s_2) - \Im(s_2)) \\ &\quad + i\left[\Im(s_1) + \frac{1}{\sqrt{2}}\left(\Im(s_2) + \Re(s_2)\right)\right] \end{aligned}$$

The decomposition of the coded symbol into real and imaginary parts leads to numbers of the form $n = a + \frac{1}{\sqrt{2}}b$ where a belongs to the Z-PAM, but b is the sum of two elements of the Z-PAM. The modified Cassels' algorithm is thus modified again to respect this constraint (see Algorithm 2).

Because of the real and imaginary decomposition, θ is replaced by $\frac{1}{\sqrt{2}}$ on lines 1, 2 and 26. Moreover, as the second element of the approximation is a sum of symbols taken in a Z-PAM, the search interval has to be changed. This is done on lines 1, 7 and 25.

The exhaustive search has a complexity of the order M^4. The two-step decoding method allows to reduce this complexity order to M^2. Finally, the combination with the diophantine approximation approach reduces the complexity order to $\sqrt[4]{M}$.

V. Simulation results

Simulations have been run for both the one-relay and two-relay cases.

In the one-relay case, the Incomplete DF has been implemented with the distributed Golden code. Both exhaustive decoding and diophantine approximation are considered at the relay.

Figure 3 represents the performance obtained for both methods. The set of upper curves have been simulated with a spectral efficiency of 4 bits per channel use, the other set with 2 bits pcu. Diophantine approximation at relay performs slightly worser than the exhaustive decoding. This is explained by the fact that it is not an optimal decoding. However, this small loss in performance, only 0.5 dB, is counterbalanced by

Input: y, Z
Output: $\widehat{X}$
1 $\beta = -(y + (Z+1) + 2Z\frac{1}{\sqrt{2}}))/2$;
2 $\alpha = -\frac{1}{\sqrt{2}}$;
3 $\eta_0 = \alpha; \eta_1 = -1; \zeta_1 = -\beta$;
4 $p_0 = 0; p_1 = 1; P_1 = 0$;
5 $q_0 = 1; q_1 = 0; Q_1 = 0$;
6 $n = 2$;
7 **while** $\eta_{n-1} \neq 0 \wedge \zeta_{n-1} \neq 0 \wedge Q_{n-1} \leq 2Z - 1$ **do**
8 $\quad a_n = \lfloor -\frac{\eta_{n-2}}{\eta_{n-1}} \rfloor$;
9 $\quad p_n = p_{n-2} + a_n p_{n-1}$;
10 $\quad q_n = q_{n-2} + a_n q_{n-1}$;
11 $\quad \eta_n = \eta_{n-2} + a_n \eta_{n-1}$;
12 $\quad$ **if** $Q_{n-1} \leq q_{n-1}$ **then**
13 $\qquad b_n = \lfloor -\frac{\zeta_{n-1} - \eta_{n-2}}{\eta_{n-1}} \rfloor$;
14 $\qquad P_n = P_{n-1} + p_{n-2} + b_n p_{n-1}$;
15 $\qquad Q_n = Q_{n-1} + q_{n-2} + b_n q_{n-1}$;
16 $\qquad \zeta_n = \zeta_{n-1} + \eta_{n-2} + b_n \eta_{n-1}$;
17 $\quad$ **else**
18 $\qquad P_n = P_{n-1} - p_{n-1}$;
19 $\qquad Q_n = Q_{n-1} - q_{n-1}$;
20 $\qquad \zeta_n = \zeta_{n-1} - \eta_{n-1}$;
21 $\quad$ **end**
22 $\quad n = n + 1$;
23 **end**
24 $P = 2P_n - (Z+1)$;
25 $Q = 2Q_n - 2Z$;
26 $\widehat{X} = P + \frac{1}{\sqrt{2}}Q$;

Algorithm 2: Modified Cassels' algorithm for $\theta = e^{i\frac{\pi}{4}}$

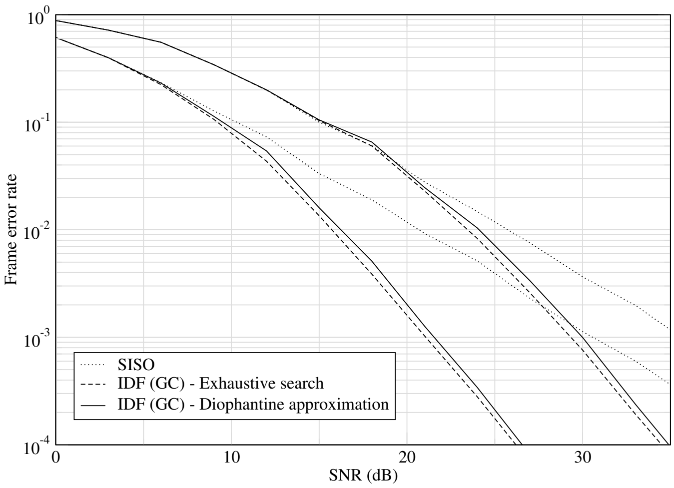

Fig. 3. Performance of the IDF protocol in the one-relay case with exhaustive search or diophantine approximation at relay. Frame error rate is plotted as function of the SNR at spectral efficiencies of 2 and 4 bits per channel use.

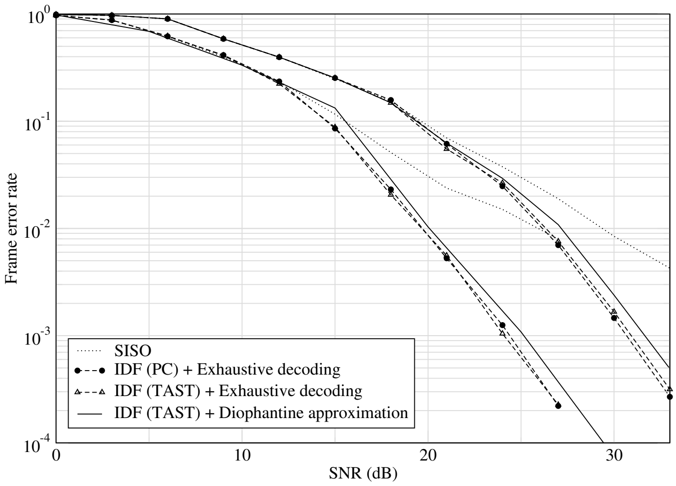

Fig. 4. Performance of the IDF protocol in the two-relay case with exhaustive search or diophantine approximation at relays. Frame error rate is plotted as function of the SNR at spectral efficiencies of 2 and 4 bits per channel use.

a much lower decoding complexity decreasing from M^2 to $\sqrt[4]{M}$.

In the two-relay case, the Incomplete DF protocol has been implemented with the distributed 4×4 perfect and TAST codes. When using the distributed perfect code, only exhaustive search is possible at relays. When using the distributed TAST code, both exhaustive search and diophantine approximation are considered.

Figure 4 represents the performance of the IDF protocol obtained by simulation. On can see that the use of the TAST code instead of a perfect code does not induce a big loss in performance, even if the TAST code does not respect the NVD property unlike the perfect code. Moreover, the diophantine approximation causes less than 1 dB loss compared to the exhaustive search, which makes the total loss equal to 1 dB compared to the distributed perfect code with an exhaustive decoding at relays.

Ideally the Incomplete DF would be the least complex and more performant if used with a distributed STBC offering both a structure allowing the two-step decoding and $\theta \in \mathbb{R}$ for a simple diophantine approximation.

VI. Conclusion

In this paper, we considered the Incomplete DF protocol previously proposed in [4]. This protocol is based on a incomplete decoding at relays, which was originally performed by an exhaustive search. This last method induces a high processing complexity, increasing with the number of relays and the size of the constellation. In this work, we showed that the decoding problem at relays can be reformulated as a diophantine approximation problem. We proposed a modification of the well-known Cassels' algorithm to the approximation of an element of $\mathbb{Q}(i, \theta)$, with $\theta \in \mathbb{R}$, by a linear combination $a + \theta b$ of two elements taken in a Z-PAM. This modified algorithm is used in the one-relay case, to decode elements of the distributed Golden code. This strategy allows a drop of the processing complexity from the order M^2 to the order $\sqrt[4]{M}$ only.

In [4], a method based on the structure of TAST codes was proposed to reduce the processing complexity when the number of relays increase. In order to combine this method and the diophantine approximation approach, an other adaptation of Cassels' algorithm is proposed with $\theta = e^{i\frac{\pi}{4}}$. In the two-relay case, the combination of these two strategies provides a decrease in processing complexity from the order M^4 to the order $\sqrt[4]{M}$ only.

Simulations show that the diophantine approximation at relay induces only a small loss in performance However, this is a low price for an important drop of the processing complexity.

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