Adaptive Amplify-and-Forward Cooperative Channel

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***Abstract*—In this work, we propose an amplify-and-forward (AF) protocol based on a new selection criterion, a function of the instantaneous capacities of all possible transmission schemes (with or without cooperation). The outage probability and the simulation results show that the new Adaptive AF protocol has better performance than the best known AF protocol (NAF). Moreover, this protocol solves the problem of the bad performance of NAF at low SNR.**

We also make a relay-selection to obtain a better diversity-order and then better performance.

I. Introduction

Diversity techniques have been developed in order to combat fading on wireless channels. Recently, a new diversity technique has been proposed with cooperative systems [1], [2]. Different nodes in the network cooperate in order to form a MIMO system array and exploit space-time diversity. Lots of cooperative protocols have been proposed [3]-[6], which can be classified in three main family: amplify-and-forward (AF), decode-and-forward (DF), and coded-cooperation (CC).

AF protocols have been studied the most due to their simplicity. This strategy consists in amplifying the received message and forwarding it at the relay. Asymptotically, it brings diversity and gives better performance than SISO, which only uses direct link. However it does not match non-cooperation at low SNR. Even the NAF protocol presented in [4], [5], associated with perfect codes [7], which is the best known AF scheme [8], gives much worse results than SISO for low SNR, especially at high spectral efficiencies.

We propose here a new selection for AF protocols, and in particular NAF protocols. Selection has already been proposed in literature for DF protocols [9], [5], but never adapted to AF protocols. The selection criterion of DF protocol is based on source-relay outage probability. We define here a new selection criterion, a function of the instantaneous capacities of all possible transmission schemes. This new selection criterion leads to the definition of an Adaptive AF protocol performing better than the NAF protocol, and than SISO for low SNR.

II. System model and NAF protocol

A. Channel model

We consider $N+1$ sources who want to transmit messages to the same destination D. The channel is shared in a TDMA manner, so each source is allocated a different time slot and the system can be reduced to a relay channel with one source, N relays and one destination. The $N+1$ sources play the role of the source in succession while the others are used as relays.

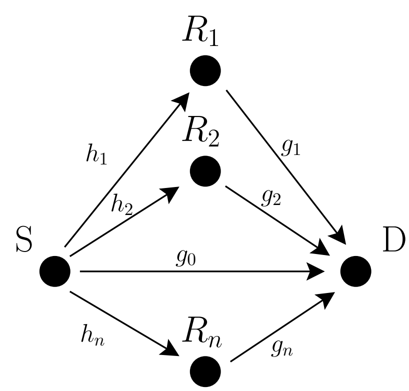


Fig. 1. System model : a relay channel with one source, N relays and one destination

We consider a half-duplex channel; each terminal, and in particular the relays, cannot receive and transmit at the same time. The channel links are Rayleigh, slow fading, so we can consider their coefficients as constant during at least the transmission of one frame.

We suppose that all terminals are equipped with only one antenna; the MIMO case is not considered in this work. We focus here on the protocol. So, for simplicity, we assume a uniform energy distribution between source and relays.

In the next sections, we will use the notation given in Figure 1. The channel coefficient of the link between source S and destination D is g_0, the one between source S and relay R$_i$ is h_i and the one between relay R$_i$ and destination D is g_i.

B. NAF protocol

In this paper, we consider the non orthogonal AF (NAF) protocol proposed in [4] for the one-relay case and generalized in [5] to $N>1$ relays.

This protocol is schematized in Table I. The source keeps transmitting: x_{11} during the first time slot and x_{12} during the second one and so on... During the first time slot, the first

TABLE I
NAF PROTOCOL

S	x_{11}	x_{12}	x_{21}	x_{22}	...	x_{N1}	x_{N2}
R_1	y_{r1}	$\beta_1 y_{r1}$					
R_2			y_{r2}	$\beta_2 y_{r2}$			
$\vdots$					$\ddots$		
R_N						y_{rN}	$\beta_N y_{rN}$
D	y_{11}	y_{12}	y_{21}	y_{22}	...	y_{N1}	y_{N2}

relay listens y_{r1}, and, during the second time slot, retransmits a scale version of the signal $\beta_1 y_{r1}$.

In the next paragraphs, we will use the same notation: x_{i1}, x_{i2} are the signals to be transmitted, y_{ri} is the received signal at the ith relay and y_{i1}, y_{i2} are the received signals at destination. β_i is the scale factor of the ith relay.

In order to better understand this protocol, we develop here the one-relay case. For simplicity of notation, when studying the one-relay case, we will use x_1 and x_2, y_r and β, y_1 and y_2 as in Table II. SNR stands for the signal-to-noise ratio.

TABLE II
NAF PROTOCOL FOR 1 RELAY

S	x_1	x_2
R	y_r	βy_r
D	y_1	y_2

The optimum value of β has been calculated in [8]:

$$\beta = \frac{1}{\sqrt{1+\mathsf{SNR}|h|^2}}$$

We can define the following received signals at relay and destination respectively.

$$\begin{aligned} y_r &= \sqrt{\mathsf{SNR}} h x_1 + v \\ y_1 &= \sqrt{\mathsf{SNR}} g_0 x_1 + w_1 \\ y_2 &= \sqrt{\frac{\mathsf{SNR}}{2}} g_0 x_2 + \sqrt{\frac{\mathsf{SNR}}{2}} g_1 \beta y_r + w_2 \end{aligned}$$

where the factor $\frac{1}{\sqrt{2}}$ comes from the equally distributed energy assumption.

We can rewrite these equations in the following way

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = &\sqrt{\mathsf{SNR}} \begin{bmatrix} g_0 & 0 \\ \sqrt{\frac{\mathsf{SNR}}{2}} g_1 \beta h & \frac{1}{\sqrt{2}} g_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ \sqrt{\frac{\mathsf{SNR}}{2}} g_1 \beta v \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \end{aligned}$$

which can be rewritten in the form

$$Y = \sqrt{\mathsf{SNR}} H X + W$$

with

$$H = \begin{bmatrix} g_0 & 0 \\ \sqrt{\frac{\mathsf{SNR}}{2+\mathsf{SNR}|g_1|^2\beta^2}} g_1 \beta h & \frac{1}{\sqrt{2+\mathsf{SNR}|g_1|^2\beta^2}} g_0 \end{bmatrix}$$

where normalization takes the variance of the noise into account.

The equivalent model $Y = HX + W$ can be calculated for any number of relays. After vectorization and rewriting of complex expressions by using the real and imaginary parts, we obtain a lattice representation of the system. So decoding can be performed by using ML lattice decoders, such as the sphere decoder or the Schnorr-Euchner algorithm.

It has been proven in [8] that this protocol is optimal when used with the Golden code [10] for the one-relay case, or a corresponding perfect code $2N \times 2N$ [7] for the N-relay case.

For the one-relay case, the NAF protocol using the Golden code is summarized in Table III.

TABLE III
NAF PROTOCOL ASSOCIATED TO THE GOLDEN CODE $\left(\alpha = \frac{1+i-i\theta}{\sqrt{5}}\right.$ AND $\left.\theta = \frac{1+\sqrt{5}}{2}\right)$

S	$\alpha(s_1+\theta s_2)$	$\alpha(s_3+\theta s_4)$	$i\overline{\alpha}(s_3+\overline{\theta}s_4)$	$\overline{\alpha}(s_1+\overline{\theta}s_2)$
R	y_{r1}	y_{r2}	βy_{r1}	βy_{r2}
D	y_1	y_2	y_3	y_4

III. NEW SELECTION PROTOCOL

The existing AF protocols bring a diversity advantage at high SNR, but their performance at low SNR are poorer than those of the SISO scheme. To solve this issue, we propose a new Adaptive AF protocol where the choice of a transmission scheme is based on the links quality.

A. Presentation of the Adaptive AF

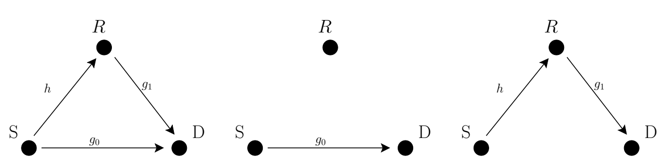
To establish the selection criterion we have to consider all possible transmission schemes. Let's begin by detailing the one-relay case before looking for the generalization.

One-relay case: There are three possible transmission schemes as presented in figure 2:

(a) NAF case: full cooperation scheme is used, symbols are sent using the NAF protocol associated with the Golden code. The symbol rate is 1 symbol per channel use (1 symb. pcu);

(b) SISO case: only direct link is used, symbols are sent over the source-destination link (non-coded) at a rate of 1 symb. pcu;

(c) NLOS case: only non-line-of-sight (NLOS) link is used (direct link is not), symbols are sent over the source-relay link in a first phase and forwarded by the relay in a second phase (non-coded). The rate is then $\frac{1}{2}$ symb. pcu. Therefore in order to have the same spectral efficiency as in the 1 symb. pcu case, we need to use a larger



(a) NAF scheme (b) SISO scheme (c) NLOS scheme

Fig. 2. 3 possible schemes in the 1-relay case

constellation. For example, if the other protocols use 16-QAM, the NLOS scheme must use 256-QAM.

When all links are in outage and so errors occur inevitably, a 4^{th} case could be added with non-transmission. We have not considered this case however as it will induce a rate decrease. Moreover, using error correcting codes, we can improve the decoding of the message.

The principle of this new selection protocol is to evaluate the qualities of the three schemes and to choose the best of them at each frame.

Generalization to the N-relay case: This selection can be generalized quite easily to N relays.

For example, for 2 relays, the number of possible schemes is seven:

(a) full cooperation: symbols are sent using the NAF protocol for 2 relays associated with a 4×4 perfect code. The symbol rate is 1 symb. pcu;
(b) cooperation with only relay R_1 or relay R_2: symbols are sent using the NAF protocol for 1 relay associated with the Golden code. The symbol rate is 1 symb. pcu;
(c) direct link: the symbol rate is 1 symb. pcu again;
(d) both NLOS links: the symbol rate is $\frac{1}{2}$ symb. pcu;
(e) NLOS link using only relay R_1 or relay R_2: the symbol rate is $\frac{1}{2}$ symb. pcu.

The number of cases grows with the number of relays. However, it does not increase complexity that much. Indeed, there are $2^{N+1}-1$ different cases for the N-relay scheme, but only a simple test is necessary to determine the best one. As some schemes are identical except for exchanging coefficients (for example, NLOS with relay R_1 or relay R_2), the decoding complexity reduces to only $2N+1$ different algorithms. So the complexity of this new selection protocol increases linearly with the number of relays, which is quite reasonable.

B. Selection criterion

We have listed all possible transmission cases in the previous subsection, we present now a criterion to determine which one to choose at each frame. We propose to study all these schemes and to select the one which has the largest instantaneous capacity. In order to better understand this strategy, we develop here the one-relay case.

SISO case: The instantaneous capacity for the SISO scheme is easy to calculate:

$$C_{\text{SISO}}(H) = \log_2\left(1 + \mathsf{SNR}|g_0|^2\right)$$

NLOS case: To calculate the instantaneous capacity, we first have to define the received signals at relay and destination respectively.

$$y_r = \sqrt{\mathsf{SNR}}hx + v$$
$$y_d = \sqrt{\mathsf{SNR}}g_1\beta y_r + w$$

The destination does not need to listen in the first time slot, as we do not use the direct link.

$$\begin{aligned} y_d &= \sqrt{\mathsf{SNR}}g_1\beta(\sqrt{\mathsf{SNR}}hx + v) + w \\ &= \mathsf{SNR}g_1\beta hx + \sqrt{\mathsf{SNR}}g_1\beta v + w \end{aligned}$$

This equation is equivalent to

$$\widetilde{y_d} = \frac{\mathsf{SNR}}{\sqrt{1+\mathsf{SNR}|g_1|^2\beta^2}} g_1\beta hx + \widetilde{w}$$

where normalization takes the variance of the noise into account.

Then the instantaneous capacity is:

$$C_{\text{NLOS}}(H) = \frac{1}{2}\log_2\left(1 + \frac{\mathsf{SNR}^2}{1+\mathsf{SNR}|g_1|^2\beta^2}|g_1|^2\beta^2|h|^2\right)$$

where the factor $\frac{1}{2}$ comes from the fact that the frame duration is 2 channel uses.

NAF case: For the NAF scheme, we can refer to the calculations in subsection II-B.

The instantaneous capacity is

$$C_{\text{NAF}}(H) = \log\left(\det\left(I + \mathsf{SNR}HH^H\right)\right)$$

$$HH^H = \begin{bmatrix} |g_0|^2 & \sqrt{\frac{\mathsf{SNR}}{2+\mathsf{SNR}|g_1|^2\beta^2}}g_0 g_1^*\beta h^* \\ \sqrt{\frac{\mathsf{SNR}}{2+\mathsf{SNR}|g_1|^2\beta^2}}g_0^* g_1\beta h & \frac{|g_0|^2+\mathsf{SNR}|g_1|^2\beta^2|h|^2}{2+\mathsf{SNR}|g_1|^2\beta^2} \end{bmatrix}$$

$$\begin{aligned} \det\left(I + \mathsf{SNR}HH^H\right) = 1 &+ \mathsf{SNR}|g_0|^2 \\ &+ \mathsf{SNR}\frac{|g_0|^2 + \mathsf{SNR}|g_1|^2\beta^2|h|^2}{2+\mathsf{SNR}|g_1|^2\beta^2} \\ &+ \mathsf{SNR}^2\frac{|g_0|^4}{2+\mathsf{SNR}|g_1|^2\beta^2} \end{aligned}$$

$$\begin{aligned} C_{\text{NAF}}(H) = \frac{1}{2}\log_2\Bigg(1 &+ \mathsf{SNR}\left(1 + \frac{1}{2+\mathsf{SNR}|g_1|^2\beta^2}\right)|g_0|^2 \\ &+ \frac{\mathsf{SNR}^2}{2+\mathsf{SNR}|g_1|^2\beta^2}\left(|g_0|^4 + |g_1|^2\beta^2|h|^2\right)\Bigg) \end{aligned}$$

Selection criterion: The selected transmission scheme is the one offering the maximum instantaneous capacity.

$$\arg \max_{i\in\{\text{SISO},\text{NLOS},\text{NAF}\}} \{C_i(H)\}$$

C. Enhancement of the Adaptive AF

Until now, we have considered schemes with N relays and proposed corresponding adaptive AF protocols. We can enhance the system performance by increasing the diversity order. This is possible by considering a system with more than N relays, which is closer to reality.

For reasons of complexity for example, we could prefer to use a N-relay scheme rather than a $N+P$-relay scheme. Then, we choose the N best relays within the $N+P$ ones. This way, we can increase diversity to $N+P$, whereas using a simple N-relay case. Indeed, the diversity is defined as the number of independent paths from the source to the destination and there are $N+P$ possible independent paths, even if only the bests of them are used for each frame.

D. Implementation constraints

The implementation of the new Adaptive AF protocol supposes that a node in the network decides which transmission scheme to use at each frame. We suppose that this node is the destination. So it has to estimate the channel coefficient g_0 of the direct link and the product channels $g_i \beta_i h_i$ for each relay R_i, calculate the instantaneous capacity of each possible transmission scheme and determine the one to be used. Then it broadcasts only $\left\lceil \log_2 \left(2^{N+1} - 1\right) \right\rceil = N + 1$ bits at both source and relays in order to inform them about its decision.

As we consider a slow fading channel, an estimation is made for several frames and so the transmission strategy remains the same. When a new estimation is made and if the strategy has to change, it is effective after a delay of one frame during which the strategy is not optimal.

IV. Outage probability

We have already defined the instantaneous capacities for the SISO, NLOS and NAF schemes in subsection III-B. The outage probability is then easy to calculate. For each scheme:

$$Pout^{(i)} = P\left\{C_i(H) < R\right\}$$

where R stands for the spectral efficiency in bits per channel use (bits pcu).

The proposed AF protocol chooses the transmission scheme that maximizes the instantaneous capacity $C_i(H)$ over i. So the instantaneous capacity of the adaptive protocol is larger than each $C_i(H)$ for a fixed channel realization H. Thus, the selection scheme is in outage iff SISO, NLOS and NAF are all in outage. So we get,

$$P_{\text{out}}^{(\text{AAF})} \leq P_{\text{out}}^{(i)} \qquad i \in \{\text{SISO}, \text{NLOS}, \text{NAF}\}\,.$$

The expressions calculated above and this last comment allow us to compute Monte Carlo simulations.

In Figure 3, we plot the outage probabilities as a function of the SNR, for the SISO protocol, the NAF protocol and our new Adaptive AF, for a one-relay scheme and a spectral efficiency of 4 bits pcu. There are two sets of curves: one where we have only one relay, and the other one where we choose the best relay within three (enhancement proposed in subsection III-C). We can immediately check that this relay-selection brings diversity. Indeed the slope of the curves is greater with the selection between 3 relays than with only one. Moreover, we can note that the Adaptive AF performs better than NAF in both cases. Indeed we obtain a 4 dB gain asymptotically for both one-relay and three-relay selection cases. Even more interesting is the fact that the Adaptive AF always performs better than SISO, even at low SNR, which was the main weakness of the NAF protocol without selection.

In Figure 4, we plot the outage probabilities as a function of the SNR for the SISO protocol, the NAF protocol and the Adaptive AF, for a two-relay scheme and a spectral efficiency of 4 bits pcu. Here again, the enhancement of the Adaptive AF over the NAF protocol is verified, as we obtain a 5 dB gain asymptotically, and solve the problem of bad performance at low SNR.

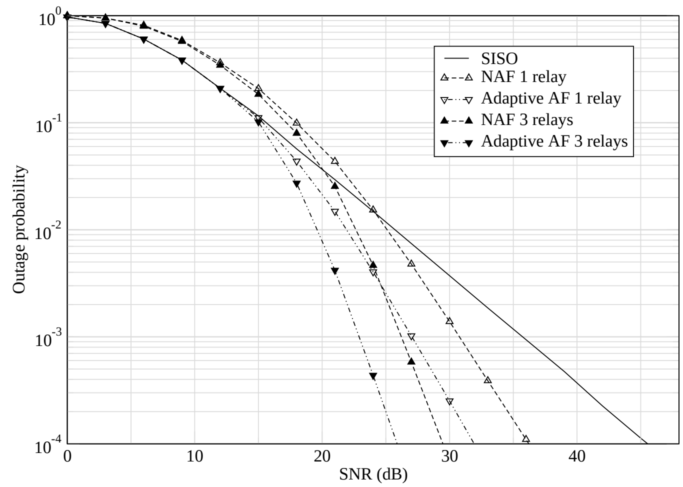


Fig. 3. 1-relay scheme: comparison of the outage probabilities of the non-cooperative case, the NAF protocol and the Adaptive AF for 4 bits pcu

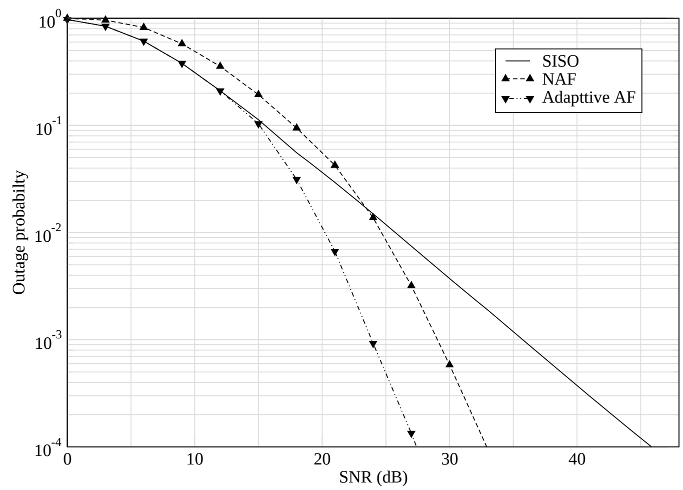


Fig. 4. 2-relays scheme: comparison of the outage probabilities of the non-cooperative case, the NAF protocol and the Adaptive AF for 4 bits pcu

V. Simulation results

In Figures 5 to 7 we plot the frame error rate of the SISO, NAF and Adaptive AF protocols as a function of the SNR.

In Figure 5 are represented the curves for a one-relay scheme. The simulation is implemented with the Golden code and a Schnorr-Euchner decoding.

There are two sets of curves, one for 2 bits spectral efficiency and the other one for 4 bits spectral efficiency. In both cases, the Adaptive AF performs better asymptotically than the NAF protocol, with a 3 dB and 4 dB gain for 2 and 4 bits spectral efficiency respectively. Moreover, in both cases again, it solves the problem of bad performance at low SNR.

Figure 6 represents the one-relay scheme with the enhancement proposed in subsection III-C, consisting in choosing the best relay among three ones.

The good performance of our new Adaptive AF are confirmed with 3 dB and 4 dB gains for 2 and 4 bits spectral efficiency respectively. Moreover, the problem of bad perfor-

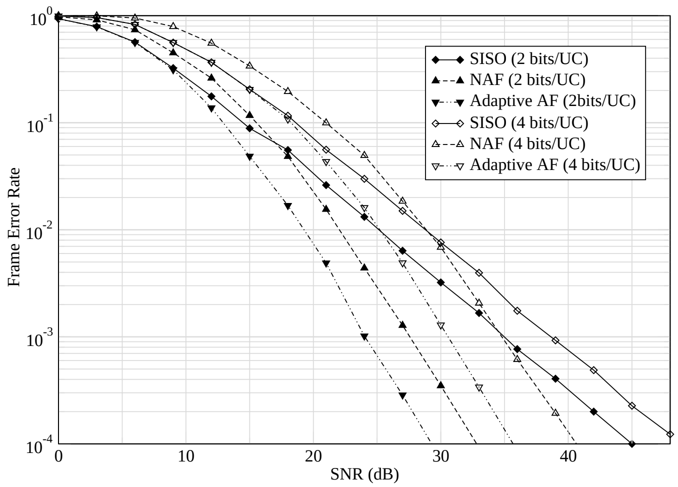


Fig. 5. 1-relay scheme: comparison of the performance of the non-cooperative case, the NAF protocol and the Adaptive AF for 2 bits pcu and 4 bits pcu

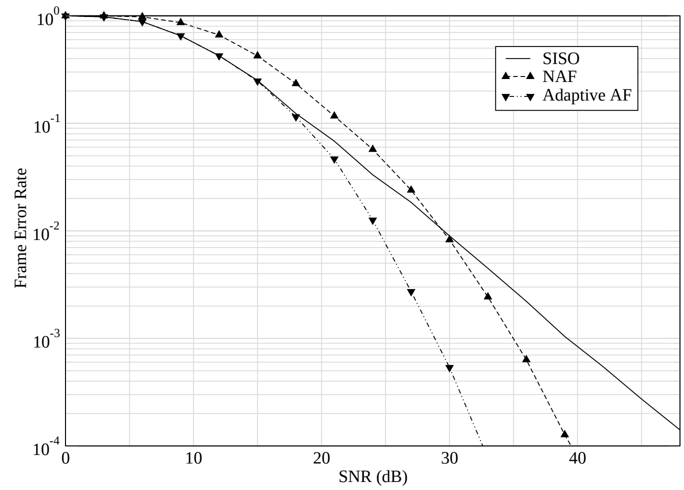


Fig. 7. 2-relays scheme: comparison of the performance of the non-cooperative case, the NAF protocol and the Adaptive AF for 4 bits pcu

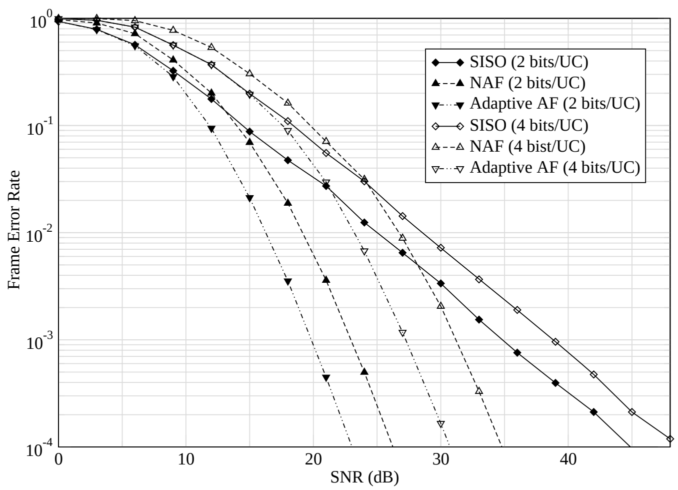


Fig. 6. 1-relay scheme: comparison of the performance of the non-cooperative case, the NAF protocol and the Adaptive AF for for 2 bits and 4 bits pcu, with 1 relay chosen out of 3

mance at low SNR is here again solved.

Besides, comparing with Figure 5, we can see the diversity gain due to the relay selection. Indeed the slope of the curves is higher with 3 relays than only one.

In Figure 7 are represented the curves for the two-relay scheme. The simulation is implemented with a 4×4 perfect code and a Schnorr-Euchner decoding, for a 4 bits spectral efficiency.

The improved performances of the Adaptive AF are here again confirmed with a 7 dB gain over the NAF protocol. Besides, the problem of bad performance of the NAF at low SNR is solved with two relays too, since the Adaptive AF curve is always under the SISO curve.

VI. Conclusion

We proposed an adaptive amplify-and-forward protocol based on a new selection criterion derived from the calculations of the instantaneous capacities of all possible transmission schemes. Both outage probability and performance from simulation results prove that the Adaptive AF enhances the performance of the NAF protocol at high SNR, and solves the problem of poor performance at low SNR.

To enhance the performance of the adaptive AF protocol, we proposed to make a relay selection which increases the diversity order and allows to obtain better asymptotic gains.

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