AF and DF Protocols based on Alamouti ST Code

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***Abstract*—In this work we propose a new amplify-and-forward (AF) protocol and a new decode-and-forward (DF) protocol based on the Alamouti space-time (ST) code, chosen because of its decoding simplicity. We also apply a new selection criterion for AF and DF protocols that improves their performance and solves the problem of bad performance at low SNR. Finally, we apply the Alamouti AF and DF protocols to a ”non-line-of-sight” (NLOS) scheme to bring diversity.**

Outage probabilities and simulation results show that at low spectral efficiency, in spite of their rate of $\frac{1}{2}$ symbol per channel use, these Alamouti AF and DF protocols have better performance than the non-orthogonal AF (NAF) protocol.

I. Introduction

Cooperation strategies [1] have been recently developed in order to exploit space-time diversity even with single-antenna terminals. These terminals cooperate in order to form a virtual MIMO array and bring ”cooperative diversity”.

AF protocols have been studied the most due to their simplicity. This strategy consists in amplifying the received message at the relays and forwarding it. DF protocols require more processing as the signals have to be decoded at the relays and then forwarded. They have interesting performance however and are even essential for multihop systems. Asymptotically, both protocols bring diversity and give better performance than SISO which only uses the direct link. However it does not match non-cooperation at low SNR (even the NAF protocol presented as Protocol I in [2] and generalized in [3] associated with Golden code [4], which is the best known cooperation scheme [5]).

In this work we propose new AF and DF protocols based on the Alamouti ST code [6]. A DF protocol using Alamouti code has already been proposed for a full-duplex channel in [7], but never in the half-duplex case. Outage probabilities and simulation results prove that the new AF and DF protocols have better performance than NAF at low spectral efficiency. Besides, we apply a new selection criterion for AF and DF protocols to improve their performance. This leads us to define an Adaptive Alamouti AF and an Adaptive Alamouti DF. And finally, we apply AF and DF Alamouti protocols to the NLOS channel where the direct link is supposed non-existant, in order to bring diversity.

II. One-relay Model

We consider a wireless network with one source, one relay and one destination. The channel is half-duplex, which means that terminals, and in particular relays, cannot receive and transmit at the same time. We assume a Rayleigh, slow fading channel, so that we can consider its coefficients as constant during the transmission of at least one frame. We suppose that all terminals are single-antenna; the MIMO case is not considered in this work. Finally, we focus on the protocol and for simplicity, we assume a uniform energy distribution between the source and the relay.

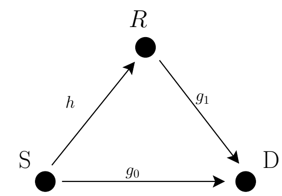

Fig. 1. Channel model: one source, one relay, one destination

In the next sections, we will use the notation given in Figure 1. The channel coefficients of the link between source and destination, source and relay and relay and destination are g_0, h and g_1 respectively.

III. Alamouti AF and DF protocols

A. Protocols

In this paper, we present new AF and DF protocols based on the Alamouti space-time code presented in [6]. These new protocols require 4 channel uses to send 2 symbols: the symbol rate is $\frac{1}{2}$ symb. pcu.

As schematized in Tables I and II, in the first phase, the source sends the first line of the Alamouti code matrix: x_1

TABLE I
Alamouti AF protocol

S	x_1	x_2	$-x_2^*$	x_1^*
R	y_{r1}	y_{r2}	βy_{r1}	βy_{r2}
D	y_1	y_2	y_3	y_4

TABLE II
Alamouti DF protocol

S	x_1	x_2	$-x_2^*$	x_1^*
R	y_{r1}	y_{r2}	$\widetilde{x_1}$	$\widetilde{x_2}$
D	y_1	y_2	y_3	y_4

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and x_2, while the relay listens. In the second phase, the relay sends either an amplified version of the received signal in the AF case ($\beta = \frac{1}{\sqrt{1+\mathsf{SNR}|h|^2}}$ being the optimal amplifying factor calculated in [5]), or a decoded version of it in the DF case, while the source sends the second line of the Alamouti code matrix: $-x_2^*$ and x_1^*.

Any DF protocol supposes that signals are correctly decoded at the relay during the first phase of the transmission. This is usually done by considering a selection between the DF and the SISO schemes with the outage event of the source-relay link as a criterion [8], [3]. Indeed, according to Shannon's theorem, if the link is in outage, no detection without error is possible: DF strategy is not efficient. In the other case, detection is possible and we use cooperation, assuming that signals have been correctly decoded. Outage probability is

$$P_O(R) = P\{\log\left(1+\mathsf{SNR}|h|^2\right) < 2R\} \tag{1}$$

where R is the global spectral efficiency and $2R$ is the one of the source-relay link.

B. Decoding

A linear decoding as for the original Alamouti ST code can be performed in both cases.

1) Alamouti AF decoding: Received signals at relay are

$$y_{r1} = \sqrt{\mathsf{SNR}}hx_1 + v_1 \text{ and } y_{r2} = \sqrt{\mathsf{SNR}}hx_2 + v_2$$

and received signals at destination

$$\begin{aligned} y_1 &= \sqrt{\mathsf{SNR}}g_0x_1 + w_1 \\ y_2 &= \sqrt{\mathsf{SNR}}g_0x_2 + w_2 \\ y_3 &= \sqrt{\tfrac{\mathsf{SNR}}{2}}(-g_0x_2^* + g_1\beta y_{r1}) + w_3 \\ y_4 &= \sqrt{\tfrac{\mathsf{SNR}}{2}}(g_0x_1^* + g_1\beta y_{r2}) + w_4 \end{aligned}$$

We can develop

$$y_3 = \sqrt{\frac{\mathsf{SNR}}{2}}\left(-g_0x_2^* + \sqrt{\mathsf{SNR}}g_1\beta hx_1\right) + \sqrt{\frac{\mathsf{SNR}}{2}}g_1\beta v_1 + w_3$$

which is equivalent to

$$\widetilde{y_3} = \sqrt{\frac{\mathsf{SNR}}{2+\mathsf{SNR}|g_1|^2\beta^2}}\left(-g_0x_2^* + \sqrt{\mathsf{SNR}}g_1\beta hx_1\right) + \widetilde{w_3}$$

where normalization takes the variance of the noise into account. Let's denote $n = \sqrt{2+\mathsf{SNR}|g_1|^2\beta^2}$.

In the same way $\widetilde{y_4} = \frac{\sqrt{\mathsf{SNR}}}{n}\left(g_0x_1^* + \sqrt{\mathsf{SNR}}g_1\beta hx_2\right) + \widetilde{w_4}$.

The system of equation can then be rewritten in the form $Y = \sqrt{\mathsf{SNR}}HX + W$ with the equivalent channel matrix H being orthogonal and the linear decoding can be performed.

2) Alamouti DF decoding: In the first phase, received signals at destination are the same, and in the second phase

$$\begin{aligned} y_3 &= \sqrt{\tfrac{\mathsf{SNR}}{2}}(-g_0x_2^* + g_1\widetilde{x_1}) + w_3 \\ y_4 &= \sqrt{\tfrac{\mathsf{SNR}}{2}}(g_0x_1^* + g_1\widetilde{x_2}) + w_4 \end{aligned}$$

where $\widetilde{x_1}$ and $\widetilde{x_2}$ are the signals decoded at relay in the considered constellation.

Assuming x_1 and x_2 have been correctly decoded, this system of equations can be rewritten with an equivalent channel matrix H being orthogonal and linear decoding can be performed once again.

IV. Performance of the Alamouti cooperation protocols

A. Outage Probability

Outage probability is defined as $P_{\text{out}}(R) = P\{C(H) < R\}$ where R is the spectral efficiency.

1) Alamouti AF case: Instantaneous capacity can be calculated from the expression of H defined in subsection III-B

$$\begin{aligned} C_{\text{ALAF}}(H) &= \tfrac{1}{4}\log\left(\det\left(I + \mathsf{SNR}HH^H\right)\right) \\ &= \tfrac{1}{2}\log\left(1 + \mathsf{SNR}\left(|g_0|^2 + \tfrac{|g_0|^2+\mathsf{SNR}|g_1|^2\beta^2|h|^2}{n^2}\right)\right) \end{aligned} \tag{2}$$

and then

$$P_{\text{outALAF}}(R) = P\{C_{\text{ALAF}}(H) < R\} \tag{3}$$

2) Alamouti DF case: We have to distinguish two cases: when we use cooperation or not.

Let's assume the source-relay link is not in outage: we use the cooperation protocol. Then

$$\begin{aligned} C_{\text{ALDF}}(H) &= \tfrac{1}{4}\log\left(\det\left(I + \mathsf{SNR}HH^H\right)\right) \\ &= \tfrac{1}{2}\log\left(1 + \mathsf{SNR}\left(|g_0|^2 + \tfrac{|g_0|^2+|g_1|^2}{2}\right)\right) \end{aligned} \tag{4}$$

and $P_{\text{outALDF}|\overline{O}}(R) = P\{C_{\text{ALDF}}(H) < R\}$.

If the source-relay link is in outage, we use the non-cooperative scheme.

$$P_{\text{outSISO}}(R) = P\left\{\log\left(1 + \mathsf{SNR}|g_0|^2\right) < R\right\} \tag{5}$$

Finally, we can write

$$P_{\text{outALDF}}(R) = P_{\text{outALDF}|\overline{O}}(R)P_{\overline{O}}(R) + P_{\text{outSISO}}(R)P_O(R) \tag{6}$$

with $P_O(R)$ defined in equation (1).

In Figure 2, we have plotted the outage probabilities of the SISO, NAF and the new Alamouti AF and DF protocols as

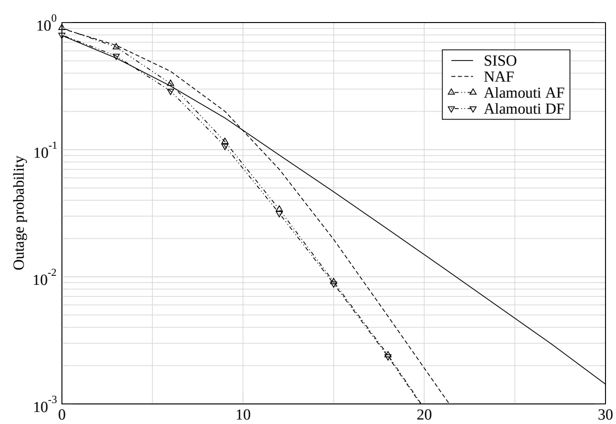

Fig. 2. Comparison of the outage probabilities of the SISO, NAF and Alamouti AF and DF protocols for 2 bits spectral efficiency

functions of the SNR (obtained numerically by Monte Carlo simulations). The spectral efficiency is 2 bits pcu. We can see that, at low spectral efficiency, the Alamouti AF and DF outperforms the NAF protocol. Moreover, the Alamouti DF has slightly better performance than the Alamouti AF for low SNR.

It is to be noticed however that, when we increase the spectral efficiency, the NAF becomes more and more performant compared to Alamouti protocols, due to rate deficiency.

B. Diversity-Multiplexing Gain Tradeoff (DMT) analysis

The DMT has been proposed in [9] in order to evaluate the asymptotic performance of space-time codes.

A diversity gain $d(r)$ is achieved at multiplexing gain r if

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log P_{\text{out}}(r\log\mathsf{SNR})}{\log\mathsf{SNR}} = -d(r)$$

Let's define

$$u_0 = -\lim_{\mathsf{SNR}\to\infty} \log_{\mathsf{SNR}} |g_0|^2 = -\lim_{\mathsf{SNR}\to\infty} \frac{\log|g_0|^2}{\log\mathsf{SNR}}$$

so we can note $|g_0|^2 \doteq \mathsf{SNR}^{-u_0}$ where $\doteq$ denotes an asymptotic behavior when $\mathsf{SNR}\to\infty$. In the same way, we define u_1 and v such as $|g_1|^2 \doteq \mathsf{SNR}^{-u_1}$ and $|h|^2 \doteq \mathsf{SNR}^{-v}$.

From the expressions of the outage probabilities (3) and (6), we can compute the DMT of our two new Alamouti AF and DF protocols.

1) DMT of the Alamouti AF: Using the asymptotic behaviors of the amplifying and normalization factors

$$\beta^2 = \frac{1}{1+\mathsf{SNR}|h|^2} \doteq \mathsf{SNR}^{-(1-v)}$$
$$n^2 = 2 + \mathsf{SNR}|g_1|^2\beta^2 \doteq \mathsf{SNR}^{1-u_1-(1-v)} \doteq \mathsf{SNR}^{-u_1+v}$$

we can write the one of the outage probability of the Alamouti AF protocol

$$\begin{aligned} &P_{\text{outALAF}}(r\log\mathsf{SNR}) \\ &\doteq P\Big\{\log\Big(\mathsf{SNR}^{1-u_0} + \tfrac{\mathsf{SNR}^{1-u_0}+\mathsf{SNR}^{2-u_1-v-(1-v)}}{\mathsf{SNR}^{-u_1+v}}\Big) \\ &\qquad\qquad < 2r\log\mathsf{SNR}\Big\} \\ &\doteq P\{\max\{1-u_0, 1-u_0+u_1-v, 1-v\} < 2r\} \\ &\doteq \mathsf{SNR}^{-d_{ALAF}} \end{aligned}$$

with

$$d_{ALAF} = \inf\{u_0+u_1+v\} = 2(1-2r) \qquad (7)$$

2) DMT of the Alamouti DF: In the same way, we can write the asymptotic behaviors of the different parts of the outage probability of the Alamouti DF protocol.

$$\begin{aligned} &P_{\text{outALDF}|\overline{O}}(r\log\mathsf{SNR}) \\ &\doteq P\left\{\log\left(\tfrac{3}{2}\mathsf{SNR}^{1-u_0} + \tfrac{1}{2}\mathsf{SNR}^{1-u_1}\right) < 2r\log\mathsf{SNR}\right\} \\ &\doteq P\{\max\{1-u_0, 1-u_1\} < 2r\} \doteq \mathsf{SNR}^{-d_1} \end{aligned}$$

with $d_1 = \inf\{u_0+u_1\} = 2(1-2r)$.

$$\begin{aligned} &P_{\text{outSISO}}(r\log\mathsf{SNR}) \doteq P\left\{\log\left(\mathsf{SNR}^{1-u_0}\right) < r\log\mathsf{SNR}\right\} \\ &\doteq P\{1-u_0 < r\} \doteq \mathsf{SNR}^{-d_2} \end{aligned}$$

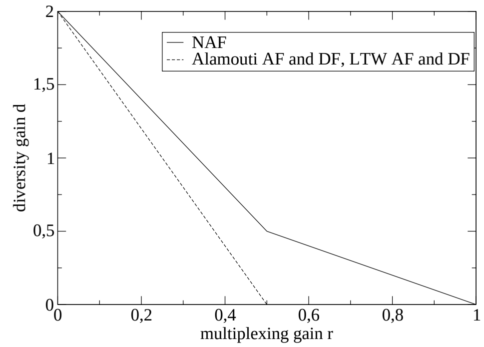

Fig. 3. Diversity-Multiplexing gain Tradeoff of the NAF and Alamouti AF and DF protocols

with $d_2 = \inf\{u_0\} = 1-r$.

$$\begin{aligned} &P_O(r\log\mathsf{SNR}) \doteq P\left\{\log\left(\mathsf{SNR}^{1-v}\right) < 2r\log\mathsf{SNR}\right\} \\ &\doteq P\{1-v < 2r\} \doteq \mathsf{SNR}^{-d_O} \end{aligned}$$

with $d_O = \inf\{v\} = 1-2r$.

Then, the asymptotic behavior of the outage probability of the Alamouti DF protocol is

$$P_{\text{outALDF}}(r\log\mathsf{SNR}) \doteq \mathsf{SNR}^{-d_{ALDF}}$$

with

$$d_{ALDF} = \min\{d_1, d_2+d_O\} = 2(1-2r) \qquad (8)$$

We can notice (Figure 3) that Alamouti AF and DF have the same DMT, lower than the one of the NAF protocol, even if they have better performance at low spectral efficiency.

C. Simulation Results

Figure 4 represents the performance of the SISO, NAF and Alamouti AF and DF protocols as functions of the SNR. The spectral efficiency is set to 2 bits pcu. We can see that both Alamouti AF and DF have better performance than NAF with a 2 dB gain asymptotically.

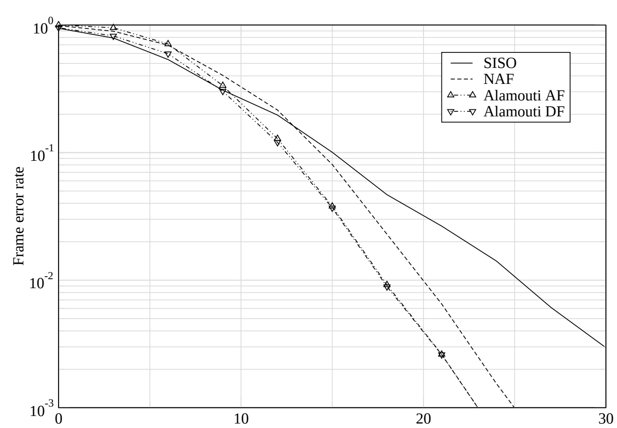

Fig. 4. Comparison of the performance of the non-cooperative case, the NAF protocol and the Alamouti AF and DF for 2 bits spectral efficiency

V. Enhancement of the Alamouti cooperation protocols: the Adaptive Alamouti AF and DF

A. Adaptive Alamouti AF

In order to improve the performance of the Alamouti AF, we propose to add a selection criterion which leads us to define a new hybrid protocol called Adaptive Alamouti AF.

We choose the transmission scheme with the highest instantaneous capacity between the SISO, NLOS AF and Alamouti AF.

SISO case: The instantaneous capacity is

$$C_{\text{SISO}}(H) = \log\left(1 + \mathsf{SNR}|g_0|^2\right) \quad (9)$$

NLOS AF case: The received signals at relay and destination respectively being $y_r = \sqrt{\mathsf{SNR}}hx + v$ and $y_d = \sqrt{\mathsf{SNR}}g_1\beta y_r + w$, we can compute the instantaneous capacity of the NLOS AF transmission scheme

$$C_{\text{NLOSAF}}(H) = \frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}^2}{1 + \mathsf{SNR}|g_1|^2\beta^2}|g_1|^2\beta^2|h|^2\right) \quad (10)$$

Alamouti AF case: Alamouti AF instantaneous capacity is given in equation (2).

B. Adaptive Alamouti DF

We also adapt this new selection criterion to DF protocols. If the source-relay link is in outage, signals cannot be decoded without error at relay, so we only use the direct link. On the contrary, if the source-relay link is not in outage, three different transmission schemes can be considered: the SISO, the NLOS DF and the Alamouti DF protocols. We choose the one with the higher instantaneous capacity.

SISO case: The instantaneous capacity of the SISO scheme is still given by equation (9).

NLOS DF case: Assuming that signal has been correctly decoded and forwarded by relay, received signal at destination is $y = \sqrt{\mathsf{SNR}}g_1\widetilde{x} + v$ where $\widetilde{x}$ is the decoded signal at relay.

And the instantaneous capacity is then

$$C_{\text{NLOSDF}}(H) = \frac{1}{2}\log\left(1 + \mathsf{SNR}|g_1|^2\right) \quad (11)$$

Alamouti DF case: Alamouti DF instantaneous capacity is given in equation (4).

VI. Performance of the Adaptive Alamouti cooperation protocols

A. Outage Probability

In the adaptive case, the instantaneous capacity is the highest between the SISO, the NLOS and the cooperation ones.

So in the AF case, the outage probability becomes

$$P_{\text{outAALAF}}(R) = P\left\{\max_{i\in\{\text{SISO,NLOS AF,AL AF}\}} C_i(H) < R\right\}$$

The system is in outage only if all schemes are in outage.

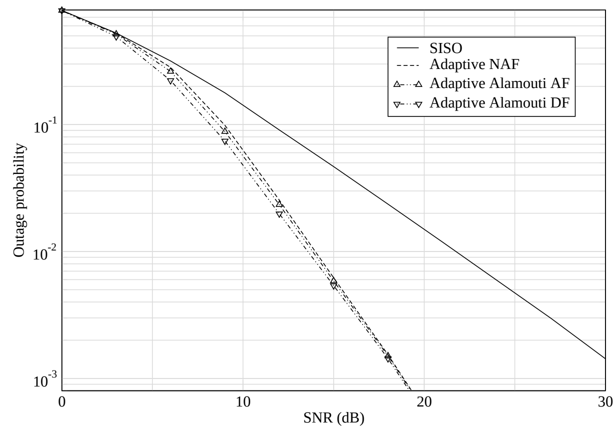

Fig. 5. Comparison of the outage probabilities of the non-cooperative case, the Adaptive NAF and the Adaptive Alamouti AF and DF for 2 bits spectral efficiency

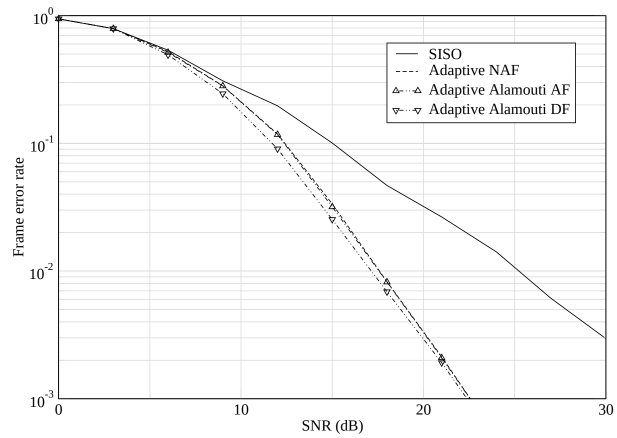

Fig. 6. Comparison of the performance of the non-cooperative case, the Adaptive NAF and the Adaptive Alamouti AF and DF for 2 bits spectral efficiency

In the DF case, the outage probability when the source-relay link is not in outage becomes

$$P_{\text{outAALDF}|\overline{O}}(R) = P\left\{\max_{i\in\{\text{SISO,NLOS DF,AL DF}\}} C_i(H) < R\right\}$$

Then, the global outage probability still is

$$P_{\text{outAALDF}}(R) = P_{\text{outAALDF}|\overline{O}}(R)P_{\overline{O}}(R) + P_{\text{outSISO}}(R)P_O(R)$$

where the other probability definitions remain the same.

On Figure 5, we plot the outage probabilities of the SISO, the Adaptive AF and the Adaptive Alamouti AF and DF as functions of the SNR. The spectral efficiency is once again set to 2 bits pcu.

We can see, that the Adaptive Alamouti AF and the Adaptive NAF have similar performances while the Adaptive Alamouti DF slightly outperforms all these schemes. Moreover adaptive schemes solve the problem of bad performance at low SNR, and increase performance with a 1 dB gain over the non-adaptive protocols.

B. Simulation Results

Figure 6 represents the performance of the SISO, Adaptive NAF and Adaptive Alamouti AF and DF protocols as functions of the SNR for a spectral efficiency of 2 bits pcu. Performance obtained with the outage probabilities calculations are confirmed. The problem of bad performance at low SNR is solved and DF protocol has slightly better performance than both AF protocols.

VII. An application of the Alamouti cooperation protocols: NLOS channel model

A. 2-relay NLOS channel

In this section, we consider a wireless network with 1 source, 2 relays and 1 destination, where the source-destination link is so bad that we do not even consider it. The other assumptions of the first part of this work are still valid. This NLOS channel model is schematized on Figure 7.

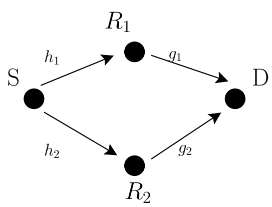

Fig. 7. Channel model: one source, two relays, one destination

B. Application of the Alamouti AF and DF to the 2-relay NLOS channel

In order to bring diversity, we propose to adapt our Alamouti AF and DF to this new channel model. The new transmission schemes are summarized in Tables III and IV.

TABLE III
Alamouti AF protocol

S	x_1	x_2		
R$_1$	y_{r1}	y_{r2}	$\beta_1 y_{r1}$	$\beta_1 y_{r2}$
R$_2$	y_{r1}	y_{r2}	$-\beta_2 y_{r2}^*$	$\beta_2 y_{r1}^*$
D			y_1	y_2

TABLE IV
Alamouti DF protocol

S	x_1	x_2		
R$_1$	y_{r1}	y_{r2}	$\widetilde{x_1}$	$\widetilde{x_2}$
R$_2$	y_{r1}	y_{r2}	$-\widetilde{x_2}^*$	$\widetilde{x_1}^*$
D			y_1	y_2

Figure 8 represents the frame error rate as a function of the SNR for the non-coded AF and DF, and the Alamouti AF and DF, at a spectral efficiency of 4 bits pcu. We can see that in the non-coded case, DF protocols are much more efficient than AF protocols. In the space-time coded case, Alamouti AF and DF have nearly the same performance, except for low SNR, where the DF protocol is more efficient.

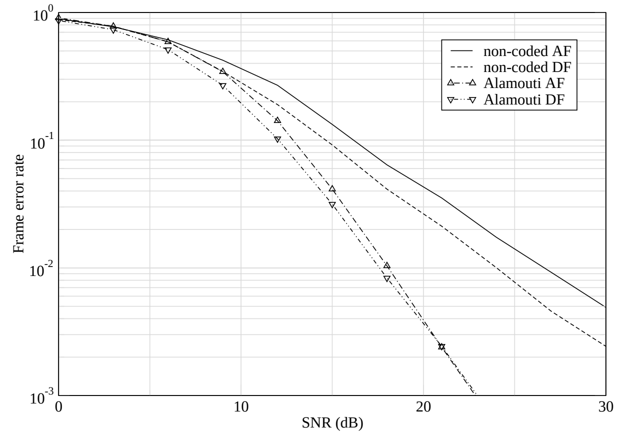

Fig. 8. Comparison of the performance of the non-coded AF, non-coded DF and Alamouti DF for 4 bits spectral efficiency

VIII. Conclusion

In this paper, we presented our work on the Alamouti space-time coded cooperation systems. We proposed two new AF and DF protocols based on Alamouti code and studied their performance in terms of outage probabilities and DMT. Moreover, we applied a new selection criterion for AF and DF protocols that improves their performance and solves the problem of bad performance at low SNR. Finally, we took interest in the case of a NLOS relay channel, and applied our Alamouti AF and DF protocols to it in order to bring diversity.

If we try to generalize this protocol to a higher number of relays, with other orthogonal codes, the symbol rate decreases dramatically. Other space-time codes have to be developed, which can be applied to DF protocols with N relays and preserve the symbol rate.

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