A Non-Quadratic Criterion for FIR MIMO Channel Equalization

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***Abstract*—In the case of memoryless MIMO channel and when the channel matrix is ill-conditioned, it is well known that performances of the Maximum Likelihood (ML) equalizer are well pronounced, compared to MMSE and ZF equalizers. In dispersive channels the conventional equalizer intends to cancel the inter-symbol interference, and did not take into the account the conditioning of the channel matrix. It intends to inverse the channel matrix somehow, which may cause noise enhancement and performances degradation. Using this fact and in order to overcome this issue, we propose in this paper a joint partial equalization and ML detection approach, where the equalizer is built based on a novel non-quadratic criterion. The proposed criterion ensures that the equalized channel matrix conserves its conditioning; which will be handled by the ML detector. Simulation results show that the improvement is well pronounced in cases where the channel matrix is ill-conditioned.**

***Index Terms*—Equalization, MIMO, detection, MMSE, conditioning, ML.**

I. INTRODUCTION

A continuous research activity has been conducted in order to combine equalization with maximum likelihood estimation (MLSE). Partial equalization consists in conditioning the channel in order to reduce the channel impulse response (CIR) order. The notion of Minimum Mean Square Error (MMSE) equalizers followed by ML detection was first conceived for single-input-single-output (SISO) transmissions [1]. Receiver and transmitter diversity allows to achieve high performance over noisy frequency-selective fading channels [2][3][4][5][6] and increases the channel capacity [7][8]. In [9], the author proposes to optimally shorten the CIR of Multiple-input-multiple-output channels in order to minimize the average energy of the error sequence between the equalized MIMO channel impulse response and a MIMO TIR with shorter memory.

In this paper we consider the case where the number of emit antennas is equal to the number of the receive antennas. We consider joint partial equalization and ML detection. The conventional MMSE consists in choosing implicitly and intuitively a one-tap Target Impulse Response TIR filter equal to identity matrix. This may cause noise enhancement in the case of ill-conditioned channel matrix. To overcome this issue, we propose a novel equalization criterion, where the TIR depends of channel matrix conditioning.

First, we introduce the proposed criterion for general number of antennas. Then, we give an implicit solution and simulation for special MIMO cases.

The rest of this paper is organized as follows; in section II, we present the input- output model and the problem formulation. The proposed approach is described in section III. In section IV, we present detailed simulation results proving that the proposed model achieves better performances.

Notation:

We adopt the following convention for notation:

- Scalars are denoted in lower cases: a.
- Vectors are denoted in lower bold cases: $\mathbf{v}$.
- Matrices are upper case bold: $\mathbf{M}$.
- First and last components of a vector are emphasized by giving them, separated by a colon, as subscript of the vector: $\mathbf{v}_{1:N}$.
- $\mathbf{I}_N$ is the identity matrix of size N.
- $trace(\mathbf{M})$ denotes the trace of the matrix $\mathbf{M}$.
- $det(\mathbf{M})$ denotes the determinant of the matrix $\mathbf{M}$.
- $\mathbb{E}\left[.\right]$ denotes the expectation operator.
- $\mathbf{O}_{N\times M}$ denotes the all- zeros matrix with N lines and M columns.
- The symbol $(^*)$ is used to denote the complex-conjugate transpose of a matrix or a vector.
- The symbol $(^t)$ is used to denote the transpose of a matrix or a vector.
- N denotes the number of emit and receive antennas.
- $\parallel . \parallel$ stands for Frobenius norm.
- δ_{ij} stands for the Kronecker index.

II. ANALYSIS

A. Input-output model

We treat a linear, dispersive and noisy channel with N emit antennas and N receive antennas. In this case, the impulse response from antenna i to antenna j is represented by a filter:

$$\mathbf{h}^{(i,j)} = \left[h_0^{(i,j)}, h_1^{(i,j)}, h_2^{(i,j)}, ...h_{L_{i,j}}^{(i,j)}\right] \qquad (1)$$

where $L_{(i,j)}$ is the channel memory between the two corresponding antennas.
The jth channel output has the following standard form:

$$y_k^{(j)} = \sum_{i=1}^{N} \sum_{m=0}^{L_{i,j}} h_m^{(i,j)} x_{k-m}^{(i)} + n_k^{(j)} \quad (2)$$

where $x_k^{i)}$ is the transmitted symbol on antenna i at time k, $y_k^{(j)}$ is the kth channel output at jth reception antenna and $n_k^{(j)}$ is the observed noise at the jth reception antenna.

The N received channel outputs may be grouped in a $N \times 1$ column vector $\mathbf{y}_k$ and can be related to the $N \times 1$ column vector of input symbols $\mathbf{x}_k$ as follows:

$$\mathbf{y}_k = \sum_{l=0}^{L} \mathbf{H}_l \, \mathbf{x}_{k-l} + \mathbf{n}_k \quad (3)$$

where $\mathbf{H}_l$ denotes the $N \times N$ lth tap of the MIMO channel response expressed as:

$$\mathbf{H}_l = \begin{bmatrix} h_l^{(1,1)} \ldots & h_l^{(N,1)} \\ \vdots & \vdots \\ h_l^{(1,N)} \ldots & h_l^{(N,N)} \end{bmatrix} \quad (4)$$

L is the longest channel memory: $L = \max_{i,j} L^{(i,j)}$.
Considering a block of N_f symbol periods, the received sequence may be expressed using the Toeplitz matrix of the channel as follows:

$$\begin{bmatrix} \mathbf{y}_{k+N_f-1} \\ \mathbf{y}_{k+N_f-2} \\ \vdots \\ \mathbf{y}_k \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \cdots & & \mathbf{H}_L & \mathbf{0} & \cdots & \\ \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & & \mathbf{H}_L & \mathbf{0} & \cdots \\ \vdots & \ddots & \ddots & & & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & & \mathbf{H}_L \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+N_f-1} \\ \mathbf{x}_{k+N_f-2} \\ \vdots \\ \mathbf{x}_{k-L} \end{bmatrix} + \mathbf{n} \quad (5)$$

or more compactly,

$$\mathbf{y}_{k+N_f-1:k} = \mathbf{H}_{channel} \, \mathbf{x}_{k+N_f-1:k-L} + \mathbf{n}_{k+N_f-1:k} \quad (6)$$

In the following, we will present the MMSE equalization approach.

B. Problem formulation: MMSE equalization

In classical Minimum Mean Square Error (MMSE) approach, the design of the optimum equalizer filter is based on the minimization of the Mean Square Error (MSE). Fig. 1 describes a block diagram of the transmission and reception scheme.

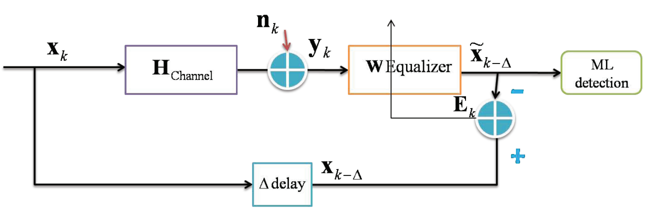

Fig. 1. Block diagram of the MIMO transmission and reception scheme

The error vector is equal to:

$$\mathbf{E}_k = \mathbf{x}_{k-\Delta} - \mathbf{W}\mathbf{y}_{k+N_f-1:k} \quad (7)$$

Where Δ is the detection delay and $\mathbf{W}$ is the MIMO equalizer filter with $(N_f \ N \times N)$ matrix taps.

The equalizer is calculated at the Training phase and is usually conceived to minimize the mean square error (MSE) given by:

$$\text{MSE} = \mathbb{E}\left[\left\|\mathbf{x}_{k-\Delta} - \mathbf{W}\,\mathbf{y}_{k+N_f-1:k}\right\|^2\right]$$

Involving the orthogonality principle, which states that $\mathbb{E}[\mathbf{E}_k \, \mathbf{y}^*_{k+N_f-1:k}] = 0$, it can be shown that the optimum equalizer in the MSE sense is given by [9]:

$$\mathbf{W}_{opt} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{R}_{\mathbf{yx}} \quad (8)$$

Where:

$$\begin{cases} \mathbf{R}_{\mathbf{xx}} = \mathbb{E}\left[\mathbf{x}_{k+N_f-1:k-L} \; \mathbf{x}^*_{k+N_f-1:k-L}\right] \\ \mathbf{R}_{\mathbf{n\,n}} = \mathbb{E}\left[\mathbf{n}_{k+N_f-1:k-L} \; \mathbf{n}^*_{k+N_f-1:k-L}\right] \\ \mathbf{R}_{\mathbf{yx}} = \mathbb{E}\left[\mathbf{y}_{k+N_f-1:k} \; \mathbf{x}^*_{k+N_f-1:k-L}\right] = \mathbf{H}\,\mathbf{R}_{\mathbf{xx}} \\ \mathbf{R}_{\mathbf{yy}} = \mathbb{E}\left[\mathbf{y}_{k+N_f-1:k-L} \; \mathbf{y}^*_{k+N_f-1:k-L}\right] = \mathbf{H}\,\mathbf{R}_{\mathbf{xx}}\,\mathbf{H}^* + \mathbf{R}_{\mathbf{nn}} \end{cases}$$

It is worth noting that a near optimal performance is met by a delay Δ equal to the equalizer order [1].
The key matrices definitions and sizes are resumed in Table 1.

After MMSE equalization is performed, ML estimation is used to detect the symbols $\mathbf{x}$.

$$\hat{\mathbf{x}}_{k-\Delta} = arg_{\mathbf{x}_{k-\Delta}} min \left\|\mathbf{x}_{k-\Delta} - \mathbf{W}_{\mathbf{opt}}\,\mathbf{y}_{k+N_f-1:k}\right\|^2 \quad (9)$$

It is clear that this conventional approach didn't take into account the conditioning of the channel matrix. Moreover, the channel matrix is somehow inverted in the equalization stage. This may engender performances degradation even though the symbols are afterward detected using ML.

To overcome this issue, we propose in this paper to introduce a memoryless target impulse response of the partial equalized channel, which preserves its conditioning.

III. PROPOSED CRITERION

In order to avoid ill-conditioned channel inversion at the equalization stage, we let the partial equalizer to compensate only dispersion effects and let a subsequent ML detection stage to deal with ill-conditioning effects. The idea is to design a one-tap TIR of the equalizer that respects channel behavior. The ML is later performed with respect to this TIR. This scheme is depicted at fig. 2. It is clear that MMSE scheme corresponds to the particular case where the TIR $\mathbf{H}_{eq}$ is the identity matrix.

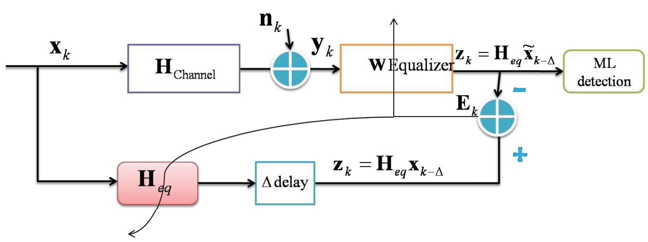

Fig. 2. Block diagram of the MIMO transmission and proposed reception scheme

By introducing this TIR, the error vector becomes:

TABLE I
KEY MATRICES INTRODUCED IN THIS PAPER

Matrix	Definition	Size
$\mathbf{R_{xx}}$	Input auto- correlation matrix	$N(N_f+L)\times N(N_f+L)$
$\mathbf{R_{nn}}$	Noise auto- correlation matrix	$NN_f \times NN_f$
$\mathbf{R_{xy}}$	Input- autput cross- correlation matrix	$N(N_f+L)\times NN_f$
$\mathbf{R_{yy}}$	Output auto- correlation matrix	$NN_f \times NN_f$
$\mathbf{W}$	Linear equalizer	$NN_f \times N$
$\tilde{\mathbf{B}}$	Augmented target impulse response filter	$N(N_f+L)\times N$
$\mathbf{R_{ee}}$	Error auto- correlation matrix	$N \times N$
$\mathbf{H}_{channel}$	Channel matrix	$NN_f \times N(N_f+L)$

$$\begin{aligned} \mathbf{E}_k = \quad & \mathbf{H_{eq}x}_{k-\Delta} - \mathbf{W}\,\mathbf{y}_{k+N_f-1:k} \\ & = \tilde{\mathbf{B}}^* \, \mathbf{x}_{k+N_f-L:k-L} - \mathbf{W}\mathbf{y}_{k+N_f-1:k} \end{aligned} \quad (10)$$

where $\tilde{\mathbf{B}} = [0_{N\times N\Delta}\, \mathbf{H}_{eq}\; 0_{N\times Nc}]$ is the augmented TIR filter and $c = N_f + L - \Delta - 1$.

The MSE is given by:

$$\begin{aligned} \text{MSE} \quad &= \quad \mathbb{E}\left[\left\|\mathbf{H_{eq}x}_{k-\Delta} - \mathbf{W}\,\mathbf{y}_{k+N_f-1:k}\right\|^2\right] \\ &= \quad \text{trace}\,(\mathbf{R_{ee}}) \end{aligned}$$

Where $\mathbf{R_{ee}}$ is the error autocorrelation matrix given by:

$$\begin{aligned} \mathbf{R_{ee}} \quad &= \mathbf{H}_{eq}(\mathbf{R_{xx}} - \mathbf{R_{xy}}\,\mathbf{R_{yy}}^{-1}\,\mathbf{R_{yx}})\,\mathbf{H}_{eq}^* \\ &= \mathbf{H}_{eq}\mathbf{S}\,\mathbf{H}_{eq}^* \end{aligned} \quad , \quad (11)$$

and

$$\mathbf{S} = \mathbf{R_{xx}} - \mathbf{R_{xy}}\,\mathbf{R_{yy}}^{-1}\,\mathbf{R_{yx}} \quad (12)$$

Note that the matrix $\mathbf{S}$ is a symmetric, positive defined matrix, which depends only on the channel conditions and not on the chosen TIR.

The optimum equalizer in the MSE sense is given in [9] by:

$$\mathbf{W}_{opt} \quad = \quad \tilde{\mathbf{B}}^*\,\mathbf{R_{xy}}\,\mathbf{R_{yy}}^{-1}$$

The main question is how to choose $\tilde{\mathbf{B}}$, (thus the Target Impulse Response $\mathbf{H_{eq}}$), in order to avoid performances degradation for ill-conditioned channels? This question will be addressed in the following sections.

A. Catastrophic solution when using MMSE criterion for TIR design

When considering a unit one norm of the TIR which minimizes the Mean Square Error (MSE), the TIR optimization problem is given by:

$$\begin{cases} \mathbf{H}_{eq}^{opt} = & arg\,min_{\mathbf{H}_{eq}}\, trace\,(\mathbf{R_{ee}}) \\ \text{Subject to:} & \|\mathbf{H_{eq}}\| = 1 \end{cases} \quad (13)$$

Which is equivalent to:

$$\begin{cases} \mathbf{H}_{eq}^{opt} = & = \arg\min trace\,\{\mathbf{H}_{eq}\mathbf{S}\,\mathbf{H}_{eq}^*\} \\ \text{Subject to:} & \|\mathbf{H_{eq}}\| = 1 \end{cases} \quad (14)$$

To solve the last equation, let's consider the eigendecomposition of the matrix $\mathbf{S}$:

$$\mathbf{S} = \mathbf{U}^*\mathbf{\Lambda}\mathbf{U} \quad (15)$$

where $\mathbf{U}$ is a unitary matrix satisfying $\mathbf{U}\,\mathbf{U}^* = \mathbf{I}$ and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_N)$ is a positive diagonal matrix containing the positive eigenvalues λ_i of $\mathbf{S}$.

As shown in [9], it is easy to prove that the optimal $\mathbf{H}_{eq}$ is composed of columns which are co-linear to the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{S}$. Therefore, the rank of $\mathbf{H}_{eq}$ is 1. Doing so, we loose the channel diversity. This leads to a catastrophic solution and performance degradation.

B. The proposed non-quadratic criterion for TIR design

The goal is to find the TIR that conserves the channel ill-conditioning features and avoid the catastrophic solution. To do so, we should ensure that the matrix $\mathbf{H}_{eq}$ has a full rank. Hence, we propose to introduce the term $\frac{\alpha}{\det(\mathbf{H}_{eq}^*\mathbf{H}_{eq})}$ in the optimization criterion, where α is a strictly positive real number.

The proposed non-quadratic criterion for channel equalization is given as follows:

$$\mathbf{H}_{eq}^{opt} = arg\,min_{\mathbf{H}_{eq}} \left(trace\,(\mathbf{R}_{ee}) + \frac{\alpha}{\det(\mathbf{H}_{eq}^*\mathbf{H}_{eq})} \right) \quad (16)$$

The proposed criterion is a compromise between minimizing the MSE and ensuring that $\mathbf{H}_{eq}$ has a full rank. In the following section, we will provide the steps to compute $\mathbf{H}_{eq}$.

IV. Optimal target impulse response determination

In this section, we will give a solution to the problem depicted in equation (16). First, let us denote $\tilde{\mathbf{H}} = \mathbf{U}\,\mathbf{H}_{eq}$. It is clear that:

$$\begin{aligned} trace(\mathbf{H}_{eq}\mathbf{S}\mathbf{H}_{eq}^*) = \quad & trace(\tilde{\mathbf{H}}\mathbf{\Lambda}\tilde{\mathbf{H}}^*) \\ \det(\mathbf{H}_{eq}^*\mathbf{H}_{eq}) = \quad & \det(\tilde{\mathbf{H}}^*\,\tilde{\mathbf{H}}) \end{aligned} \quad (17)$$

The optimization problem (16) becomes minimizing the cost function J given by:

$$J = trace(\tilde{\mathbf{H}}^*\,\Lambda\,\tilde{\mathbf{H}}) + \frac{\alpha}{\det(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^*)} \quad (18)$$

Using the fact that for square matrices $\mathbf{A}$ and $\mathbf{B}$, $trace(\mathbf{AB}) = trace(\mathbf{BA})$, the last optimization problem becomes:

$$J = trace(\mathbf{G}\Lambda) + \frac{\alpha}{\det(\mathbf{G})}, \quad (19)$$

where,

$$\mathbf{G} = \tilde{\mathbf{H}}\,\tilde{\mathbf{H}}^* \quad (20)$$

$\mathbf{G}$ is symmetric, definite and positive matrix. Let $\mathbf{v}_i$, $i = 1..N$, be the eigenvectors of $\mathbf{G}$ with the corresponding eigenvalues β_i. Due to the nature of $\mathbf{G}$, the vectors $\mathbf{v}_i$ form an orthonormal basis ($\mathbf{v}_i^*\mathbf{v}_j = \delta_{ij}$) and β_i are positive values.

By finding $\mathbf{G}$ which minimizes J, we can solve equation (20) and determine $\tilde{\mathbf{H}}$.

Equation (20) has an infinite solutions, one of these solution is choosing $\tilde{\mathbf{H}}$ as a symmetric, positive and defined matrix with eigenvectors $\mathbf{v}_i$ and eigenvalues $\sqrt{\beta_i}$. In the following, we will focus on solving equation (19) and determine $\mathbf{v}_i$ and β_i.

A. Determination of β_i

Recalling that for a square matrix $\mathbf{M}$ with dimension N, we have:

$$trace(\mathbf{M}) = \sum_{k=1}^{N} \mathbf{e}_k^* \mathbf{M} \mathbf{e}_k \tag{21}$$

where $\{\mathbf{e}_k\}$ is an orthonormal basis.

Applying this property by using the orthonormal basis $\{\mathbf{v}_i\}$ and the fact $\mathbf{v}_i$ are the eigenvectors $\mathbf{G}$, the optimization problem given in equation (19) becomes:

$$J = \sum_{i=1}^{N} \beta_i \rho_i + \frac{\alpha}{\prod_{i=1}^{N} \beta_i} \tag{22}$$

Where:

$$\rho_i = \mathbf{v}_i^* \Lambda \mathbf{v}_i$$

Assume that we know the eigenvectors $\mathbf{v}_i$, to determine the eigenvalues β_i which minimizes J, we set all the partial derivatives of J respecting to β_i to zero:

$$\frac{\partial J}{\partial \beta_i} = \rho_i - \frac{\alpha}{\beta_i \prod_{k=1}^{N} \beta_k} = 0 \tag{23}$$

leads to:

$$\beta_i = \frac{1}{\rho_i} \alpha^{\frac{1}{N+1}} \left(\prod_{k=1}^{N} \rho_k \right)^{\frac{1}{N+1}} \tag{24}$$

Equation (24) shows that β_i depends on $\mathbf{v}_i$.

B. Determination of orthonormal basis $\{\mathbf{v}_i\}$

Replacing the value of β_i in equation (22) leads to:

$$J = (N+1)\, \alpha^{\frac{1}{N+1}} \left(\prod_{i=1}^{N} \mathbf{v}_i^* \mathbf{\Lambda} \mathbf{v}_i \right)^{\frac{1}{N+1}} \tag{25}$$

Finding the orthonormal basis which minimizes J is finding the orthonormal basis $\{\mathbf{v}_i\}$ which minimizes the product:

$$P = \prod_{i=1}^{N} \mathbf{v}_i^* \mathbf{\Lambda} \mathbf{v}_i \tag{26}$$

To solve this optimization problem, we will use the following lemma:

Lemma: For a symmetric, definite and positive matrix $\mathbf{\Lambda}$ with normalized eigenvectors $\mathbf{u}_i$ and corresponding eigenvalues λ_i, such that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$, the set of orthonormal vectors $\{\mathbf{v}_1, \mathbf{v}_2, \cdots \mathbf{v}_k\}$ which minimizes P_k defined by:

$$P_k = \prod_{i=1}^{k} \mathbf{v}_i^* \Lambda \mathbf{v}_i, \text{ for } k \leq N \tag{27}$$

is given by:

$$\mathbf{v}_i = e^{(j\theta_i)} \mathbf{u}_i, \tag{28}$$

where θ_i is an arbitrary angle and $j^2 = -1$.

Proof: Due to the lack of space the proof of this lemma will not be provided in this paper ■

Using the result of this Lemma, we show that the orthonormal basis which minimizes J corresponds to the eigenvectors of Λ. Doing so, we can show easily that the terms ρ_i are equal to λ_i, and we can express the β_i as follows:

$$\beta_i = \frac{1}{\lambda_i} \alpha^{\frac{1}{N+1}} \left(\prod_{k=1}^{N} \lambda_k \right)^{\frac{1}{N+1}} \tag{29}$$

It is interesting to note from equation (29), that conditioning of the matrix $\mathbf{G}$ is equal to the conditioning of the matrix $\mathbf{S}$, which means that the proposed non-quadratic criterion allows us to conserve the initial conditioning of the channel matrix. Therefore, in the case of ill-conditioned channel, the proposed TIR conserves ill-conditioning effects which will be treated in ML detection stage.

V. Numerical Simulations

We apply the novel joint equalization and ML detection scheme over a 2×2 transmission. The CIR used in our first numerical simulations is a 2-taps filter mentioned in [9]. It describes an ill- conditioned channel and is given by the following impulse response filters:

$$\begin{aligned}
\mathbf{h}^{(1,1)} &= [\ 0.7809 \quad 0.6247\ ] \\
\mathbf{h}^{(1,2)} &= [\ 0.8945 \quad -0.4472\ ] \\
\mathbf{h}^{(2,1)} &= [\ 0.7809 \quad -0.6247\ ] \\
\mathbf{h}^{(2,2)} &= [\ 0.9579 \quad 0.2874\ ]
\end{aligned}$$

Fig. 3 depicts the variation of Bit Error Rate (BER) function of the input Signal to Noise ratio (SNR). The length of the equalizer is equal to 3. Our method succeeds in minimizing the BER with 1 dB at $BER = 10^{-6}$.

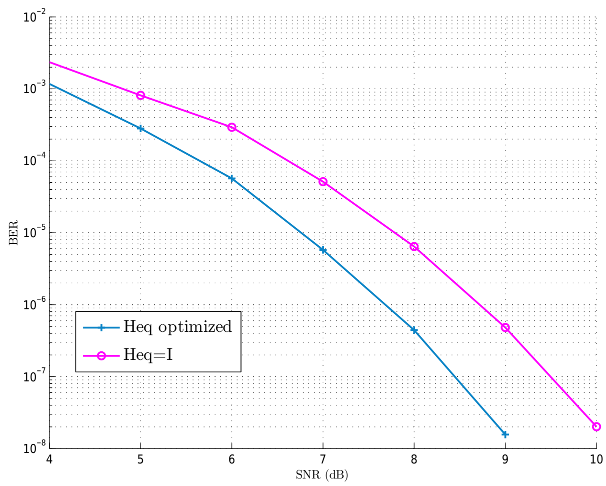

Fig. 3. Comparison of BER between conventional MMSE and proposed method

A. Channel conditioning effect

We apply the novel joint equalization and decoding scheme over a 2×2 correlated channel. The CIR used in our numerical simulations is a 2- taps filter. It describes an ill- conditioned channel with 2- taps memory given by:

$$\mathbf{h}^{(1,1)} = \begin{bmatrix} 0.8945 & 0.6247 \end{bmatrix}$$
$$\mathbf{h}^{(1,2)} = \begin{bmatrix} \omega & -0.4472 \end{bmatrix}$$
$$\mathbf{h}^{(2,1)} = \begin{bmatrix} \omega & -0.6247 \end{bmatrix}$$
$$\mathbf{h}^{(2,2)} = \begin{bmatrix} 0.8945 & 0.2874 \end{bmatrix}$$

The correlation between the two antennas is determined by a constant ω.

Fig. 4 depicts the BER of the channel conditioning function of ω value varying from 0 to 2. The SNR is fixed to 10 dB. The figure shows that a higher channel conditioning engenders a BER increase.

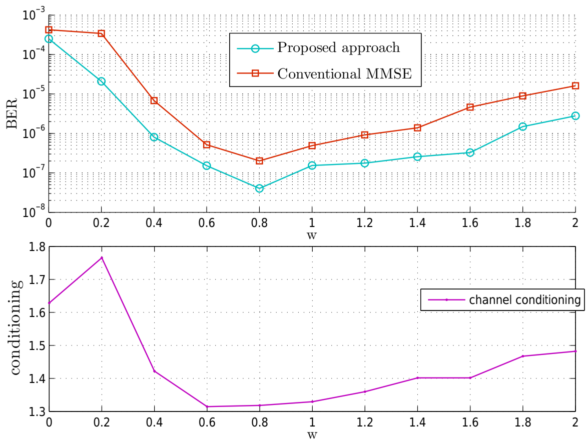

Fig. 4. Channel conditioning influence on proposed method gain at SNR=10 dB

B. Simulation for a 4×4 MIMO channel

The proposed scheme is applied to a 4×4 frequency selective and ill- conditioned MIMO channel. We treat the channel defined as follows in frequency domain:

$$\mathbf{H}(w) = \begin{bmatrix} 0.63e^{-2iwT_s} & 0 & 0 & 0 \\ 0 & 2.5e^{-2iwT_s} & 0 & 0 \\ 0 & 0 & 0.63e^{3iwT_s} & 0 \\ 0 & 0 & 0 & 2.5e^{3iwT_s} \end{bmatrix} \times \begin{bmatrix} Rotation\,(\pi/3) & \\ & Rotation\,(\pi/3) \end{bmatrix}$$

w is the frequency and T_s is the symbol time. This channel is frequency selective and ill- conditioned. Its conditioning is equal to 4.

In fig. 5, we present a the evolution of the BER versus the SNR for the proposed method and the conventional MMSE Equalizer. We can note that the proposed approach can provide a 4dB improvement at $BER = 10^{-4}$.

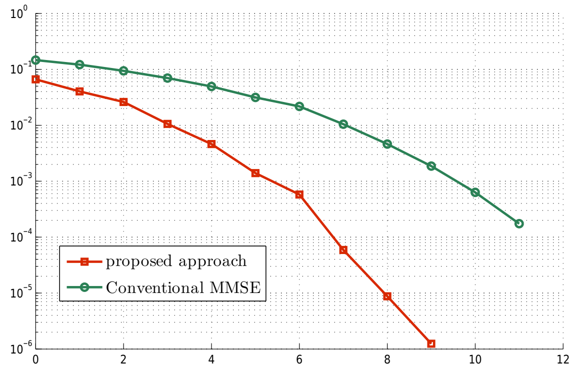

Fig. 5. Comparison of the proposed approach with the MMSE in a specific 4x4 MIMO channel

VI. Conclusion

In this paper, we derived a joint linear partial equalization and detection scheme useful for frequency selective ill-conditioned MIMO channels case. A novel non-quadratic criterion is introduced for the design of the target impulse response of the equalizer. After partial equalization, ML detection is performed. The benefit of our approach comes from non-inverting the ill-conditioned channel matrix and thus the reduction of noise enhancement. We have shown analytically that the proposed TIR matches channel conditioning and have proven by simulation that it provides better performances.

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